KPU International Journal of Engineering & Technology
Volume 5, Issue 1, Mar. 2025
e-ISSN:2790-0819, p-ISSN:2790-0800

Date Received: 2024-04-30
Date Accepted: 2025-03-09
Date Published: 2025-03-30

Numerical Solution of the Second Type of Abel's Integral Equation by Using Reproducing Kernel Space Method

MEHRULLAH MEHR* & HAJI AHMAD JOYA

Department of Mathematics, Baghlan University, Baghlan, Afghanistan.
Email: dr.mehr@baghlan.edu.af, h.ajoya@baghla.edu.af.

Abstract

It is obvious that many problems in Engineering and Science are converted to integral equations. Therefore, the principles of modeling various phenomena in engineering are based on integral equations. The objective of this study is to implement the Reproducing Kernel Hilbert Space Method on five new examples of the integral equation in order to obtain convergence as well as show the stability of the issues raised for the development of Abel's integral equations. Therefore, the stability of methods is investigated by adding perturbation in the equation. In this study, the errors of the desired problems have been obtained and compared. The graphs of the errors and the graphs of the exact solution versus the numerical solution are shown in the figure.

Keywords: Second Type of Abel's Integral Equation, Generalized, Integral Equation, Reproducing Kernel Space Method

* Corresponding Author

1. Introduction

Integral equations are considered an interesting and practical subject that is used in many Physical and Engineering phenomena such as electrostatics, Optimization, Biomechanics, Mechanics, Electrodynamics, Game Theory, Electrical Engineering, Heat Transfer, and Fluid dynamics. Furthermore, it is also used for modeling various problems which usually cannot be analyzed. Therefore, we need numerical methods to obtain the approximation of the solution with high accuracy and check the errors. One of the powerful methods with high accuracy is the method of Iterative Reproducing Kernel Hilbert Space Method. A new approach of this method for the approximate solution of nonlinear integral equations of the second type is proposed in (Javan et. al. 2017). For the convergence of the proposed method and the efficiency of the method, four examples caused by electromagnetics using the given algorithm have been compared as approximate solutions and their errors. An important type of equation is called Abel's integral equation. This equation was later extensively developed and divided into the Fredholm and Volterra equations. This equation exists in many fields of engineering sciences, mathematical modeling, astrophysics, seismology, and solid mechanics and widely in other applied sciences. Researchers have investigated the mentioned equation in various numerical methods. To solve Abel's fractional generalized integral equation using the Lager polynomial, a new method has been presented by researchers in (Singha and Nahak 2019). Singha and Nahak (2019), have used Chebyshev polynomial to reduce errors. A combination of Taylor's series and the reproducing Kernel Hilbert Space method has been used to solve Abel's integral equations numerically, and the absolute errors in two spaces have been investigated with examples.

According to Mondal and Mandal (2018), for the numerical solution of Abel's singular integral equation, the fractional differential transformation method is used and the reliability and efficiency of the method are presented with many examples. The homotopy perturbation transformation method has been used to solve Abel's integral equation, and analytical and approximate solutions of linear and nonlinear problems that appear in Engineering Sciences have been implemented by researchers (Kumar et al. 2015). In addition, for Abel's integral equation, the theorem of existence and smoothness is presented in (Atkinson 1974). Furthermore, an approximate method based on the collocation method for solving generalized Abel integral equations is presented and the convergence analysis of the method is also studied (Pandey et al. 2016). Moreover, two quadratic and cubic approximation methods are presented for solving Abel's integral equations using Caputo and Riemann-Liouville fractional derivatives and error convergence is obtained for the mentioned methods. Researchers have also compared the numerical simulations with experimental examples and numerical results for approximate designs (Kumar et al. 2019). The coupled system of Abel's integral equations of the second type with two-dimensional non-smooth solutions has been calculated by the authors in (Abdelkawy and Ameen 2019), where two-dimensional systems based on spectral discretization have been developed. An algorithm using the generative Reproducing Kernel Hilbert Space method for solving Fredholm's integral equations has been proposed and studied in two absolute error spaces (Alvandi and Paripour 2018).

Recently, integral equations have been studied by many authors in order to solve the integral equations of the second type from the homotope perturbation method, approximate and exact solutions are provided using the Laplace transform (Thota 2022). Lagrange's barycentric interpolation method for solving Volterra integral equations of the second type is presented by researchers in (Shoukralla et al. 2022). Recent developments show that the Reproducing Kernel Hilbert Space method is a powerful and stable numerical method for solving various integral equation problems that have been significantly used by researchers (Abdollahi and Abbasbandy 2022; Mehr 2023).
In a study conducted by (Mehr 2023), Abel's integral equation was examined and tested using the numerical method of Reproducing Kernel Hilbert Space while this paper intended to implement a scheme using this method on five new numerical examples, and Absolute errors, relative error and root mean square error of integral equations are obtained and compared.
The problems presented in this research are the numerical solutions. These solutions are very useful for the development and expansion of integral equations. It is because of the acceptable numerical results with negligible error, that very accurate and convergent solutions with accurate answers have been provided.

2. An Overview of Reproducing Kernel Space Method

The Reproducing Kernel Space method is a suitable numerical method for the numerical solutions of integral equations because this method has high computational accuracy. Therefore, it has been widely applied by researchers for various science issues. The original idea of this method was first developed by a PhD student named Minggen Cui studying at the Harbin Institute of Technology in the 1980s (Javan et. al. 2017 & Cui and Lin 2009). The theoretical foundations of the mentioned method are discussed below.

The Reproducing Kernel Space in the space $W_2^1[a,b]$, which is the

Reproducing Kernel of $R_y(x)$, thus,

$$R_y(x)=\begin{cases}1-a+x, & x\le y,\\ 1-a+y, & x>y,\end{cases} \tag{2.1}$$

Now we examine Abel's integral equation as follows:

$$u(x)=f(x)+Gu(x), \tag{2.2}$$

So that,

$$Gu(x)=\int_0^x \frac{u(t)}{\sqrt{x-t}}dt, \tag{2.3}$$

Then solve (2.2) in the Reproducing Kernel Space $W_2^1[a,b]$ assuming that $\psi_i(x)=R_{x_i}(x)$, so $\{x_i\}_{i=1}^{\infty}$ is dense in the interval $[a,b]$. Can write:

$$\langle u(x),\psi_i(x)\rangle_{W_2^1}=u(x_i) \quad . \tag{2.4}$$

Theorem 2.1. If $\{x_i\}_{i=1}^{\infty}$ is dense in the interval $[a,b]$, then $\{\psi_i(x)\}_{i=1}^{\infty}$ is a complete functional system of $W_2^1[a,b]$ (Cui and Lin 2009).

Proof. Given that $\{x_i\}_{i=1}^{\infty}$ is dense in the interval $[a,b]$. For

$u(x) \in W_2^1[a,b]$, if $\langle u(x), \psi_i(x) \rangle_{W_2^1} = u(x_i) = 0 \quad (i = 1,2,...)$.

As a result, we have the density of $\{x_i\}_{i=1}^{\infty}$ and the continuity of $u(x)$

$u(x) \equiv 0$.

Definition 2.2. We introduce the Gram-Schmidt orthogonality for $\{\psi_i(x)\}_{i=1}^{\infty}$ as follows (Javan et. al. 2017).

$$\overline{\psi}_i(x) = \sum_{k=1}^{i} \beta_{ik} \psi_k(x).$$

So that β_{ik} is the Gram-Schmidt orthogonalization coefficients and

$\{\overline{\psi}_i(x)\}_{i=1}^{\infty}$

Is the normal orthogonal base in $W_2^1[a,b]$ space.

Theorem 2.3. If $\{x_i\}_{i=1}^{\infty}$ is dense in the interval $[a,b]$(Cui and Lin 2009). If equation (2.2) has a unique solution, then its solution applies to the following equation.

$$u(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \left(f(x_k) + Gu(x_k) \right) \overline{\psi}_i(x) \qquad (2.5)$$

Proof. Suppose the answer is equation (2.2). From Theorem 2.1, we know that $u(x)$ is a Fourier series expansion, we have that.

$$\begin{aligned} u(x) &= \sum_{i=1}^{\infty} \left\langle u(x), \overline{\psi}_i(x) \right\rangle_{W_2^1} \overline{\psi}_i(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \left\langle u(x), \psi_k(x) \right\rangle_{W_2^1} \overline{\psi}_i(x), \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \left\langle f(x) + Gu(x), \psi_k(x) \right\rangle_{w_2^1} \overline{\psi}_i(x), \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \left(f(x_k) + Gu(x_k) \right) \overline{\psi}_i(x). \end{aligned}$$

For more information about the Reproducing Kernel Space method refer to (Mehr 2023; Cui and Lin 2009).

3. Implementations of the Method

Here is the method of solving (2.5) from (2.2) which is reproduced in a kernel space. We rewrite equation (2.5) as below (Cui and Lin 2009; Mehr 2023).

$$u(x)=\sum_{i=1}^{\infty} A_i \overline{\psi}_i(x). \tag{3.1}$$

So that

$$A_i=\sum_{k=1}^{i}\beta_{ik}\left(f(x_k)+Gu(x_k)\right). \tag{3.2}$$

Actually A_i is unknown. A_i Is obtained by known B_i. For numerical calculation, suppose the initial function is $u_1(x)=f(x)$ and the *n*-term approximation to $u(x)$ as defined below,

$$u_{n+1}(x)=\sum_{i=1}^{n} B_i \overline{\psi}_i(x), \tag{3.3}$$

So that,

$$\begin{aligned}
B_1 &= \beta_{11}\left(Gu_1(x_1)+f(x_1)\right),\\
u_2(x) &= B_1\overline{\psi}_1(x),\\
B_2 &= \sum_{k=1}^{2}\beta_{2k}\left(Gu_2(x_k)+f(x_k)\right),\\
u_3(x) &= B_1\overline{\psi}_1(x)+B_2\overline{\psi}_2(x), \qquad (3.4)\\
&\cdots\\
B_n &= \sum_{k=1}^{n}\beta_{nk}\left(Gu_n(x_k)+f(x_k)\right),\\
u_{n+1}(x) &= \sum_{k=1}^{n} B_k \overline{\psi}_k(x).
\end{aligned}$$

In this research, for the effectiveness and accuracy of the numerical method of Reproducing Kernel Hilbert Space, we consider five examples to develop the solution of integral equations. Where

$x\in[0,1], u(x),\ f(x)\in W_2^1[0,1]$.

Our goal is to solve the integral equation and get its errors. Here we use the following three criteria to measure the errors and achieve the efficiency, accuracy, and convergence of the numerical method:

$$\text{Absolute error}=\left|u(\text{exact})-u(\text{app})\right| \tag{3.5}$$

$$\text{Relative error}=\left|\left(u(\text{exact})-u(\text{app})\right)/u(\text{exact})\right| \tag{3.6}$$

$$\text{RMS error}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left|u_i(\text{exact})-u_i(\text{app})\right|^2} \tag{3.7}$$

Where N is the number of grid points and $u(\text{exact}), u(\text{app})$ shows the exact solution and approximate solution in absolute error, relative error, and root mean square error,

respectively. In the following, various examples have been calculated using Mathematica 11.3 with a computer that has a CPU capacity (1.73 GHz) and processor (Intel(R) Core (TM) i7).

Remark 1. To show the stability of the proposed methods, we apply a perturbation of size $\varepsilon = 10^{-3}$ to the right side of the equation.

4. Numerical results

Example 4.1: Consider a nonlinear Abel integral equation with the exact solution $u(x) = \frac{1}{1000} x^2$.

$$u(x) = \frac{1}{1000} x^2 \left(1 - \frac{3\pi}{16000} x^2\right) + \int_0^x \frac{u^2(t)}{\sqrt{x^2 - t^2}} dt, \qquad 0 \le x \le 1.$$

We choose $N = 20$ points and using part 3 we get an approximate solution to the problem. In this example, the absolute, relative, root mean square errors and the approximate solution of the equation are reported in Table 1, and their error plots are compared in Figure 3. The exact solution and numerical solution of $u(x)$ function is presented in Figure 1.

Table 1. Comparison of error convergence accuracy for example 4.1

Node	True solution $u(x)$	Approximate solution	Absolute error	Relative error	RMS error
0.1	0.00001	0.00001	1.96353E-11	1.96353E-6	5.92025E-12
0.2	0.00004	0.0000400001	8.044E-11	2.011E-6	2.42536E-11
0.3	0.00009	0.0000900002	1.88378E-10	2.09309E-6	5.67980E-11
0.4	0.00016	0.00016	3.44515E-10	2.15322E-6	1.03875E-10
0.5	0.00025	0.000250001	5.49454E-10	2.19782E-6	1.65667E-10
0.6	0.00036	0.000360001	8.03614E-10	2.23226E-6	2.42299E-10
0.7	0.00049	0.000490001	1.10731E-9	2.25982E-6	3.33868E-10
0.8	0.00064	0.000640001	1.46083E-9	2.28255E-6	4.40456E-10
0.9	0.00081	0.000810002	1.86436E-9	2.30168E-6	5.62127E-10

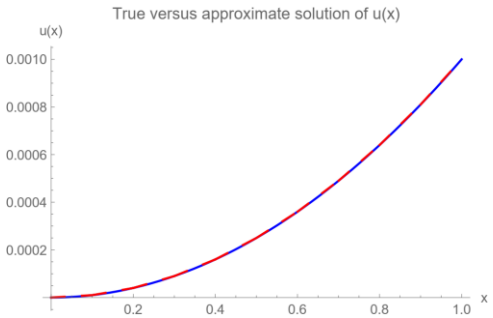


Fig. 1. The exact and numerical solutions curves for example 4.1, the continuous blue line is the exact solution, and the dashed red line is the numerical solution.

Example 4.2: Consider the flowing Abel integral equation as

$$u(x)=\sqrt{x}-\frac{x\Gamma(3/4)^2}{50(\pi)^{3/2}}+\frac{2}{100\pi}\int_0^x\frac{u(t)}{\sqrt[4]{x^2-t^2}}dt,\qquad 0\le x\le 1.$$

With the exact solution $u(x)=\sqrt{x}$. We choose $N=20$ points.

The absolute, relative, and root mean square errors and the approximate solution of the equation are reported in Table 2, and their error plots are compared in Figure 3. The exact solution and numerical solution of $u(x)$ function is

presented in Figure 2.

Table 2. Comparison of error convergence accuracy for example 4.2

Node	True solution $u(x)$	Approximate solution	Absolute error	Relative error	RMS error
0.1	0.31623	0.31611	1.1509E-4	3.63959E-4	3.47021E-5
0.2	0.44721	0.44713	8.3175E-5	1.85986E-4	2.50783E-5
0.3	0.54772	0.54765	6.9397E-5	1.26701E-4	2.09240E-5
0.4	0.63246	0.63240	6.0965E-5	9.63941E-5	1.83816E-5
0.5	0.70711	0.70705	5.5090E-5	7.79086E-5	1.66102E-5
0.6	0.77460	0.77455	5.0684E-5	6.54324E-5	1.52817E-5
0.7	0.83666	0.83661	4.7217E-5	5.64352E-5	1.42365E-5
0.8	0.89443	0.89438	4.4395E-5	4.96350E-5	1.33856E-5
0.9	0.94868	0.94864	4.2038E-5	4.43120E-4	1.26750E-5

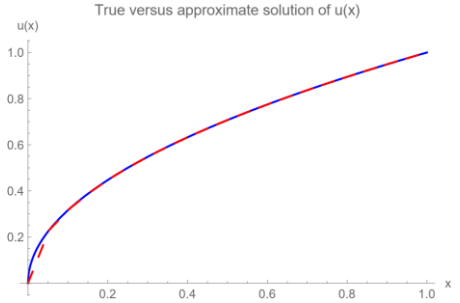
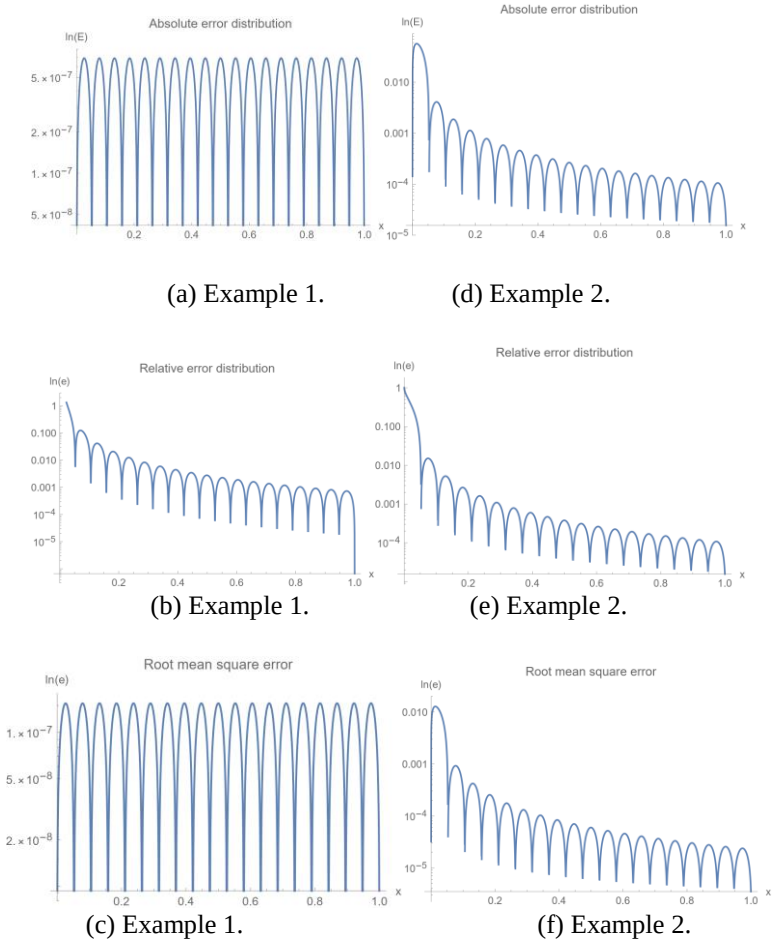


Fig. 2. The exact and numerical solutions curves for example 4.2, continuous blue line is the exact solution and dashed red line numerical solution.

Table 3. Comparison of error convergence accuracy for example 4.3

Node	True solution $u(x)$	Approximate solution	Absolute error	Relative error	RMS error
0.1	0.0031623	0.0031617	5.40683E-7	1.70979E-4	1.63022E-7
0.2	0.0044721	0.0044717	4.42623E-7	9.89734E-5	1.33456E-7
0.3	0.0054772	0.0054768	3.92849E-7	7.17240E-5	1.18448E-7
0.4	0.0063246	0.0063242	3.59913E-7	5.69072E-5	1.08518E-7
0.5	0.0070711	0.0070707	3.35783E-7	4.74868E-5	1.01242E-7
0.6	0.0077460	0.0077457	3.17014E-7	4.09263E-5	9.55833E-8
0.7	0.0083666	0.0083663	3.01821E-7	3.60746E-5	9.10026E-8
0.8	0.0089443	0.0089430	2.89167E-7	3.23298E-5	8.71870E-8
0.9	0.0094868	0.0094866	2.78396E-7	2.93455E-5	8.39394E-8



(a) Example 1. (d) Example 2.

(b) Example 1. (e) Example 2.

(c) Example 1. (f) Example 2.

Fig. 3. Figures (a), (b), (c) Error graphs for example 1, Figures (d), (e), (f) Error Graphs for example 2.

Example 4.3: Consider the nonlinear Abel integral equation with the exact solution $u(x)=\frac{1}{100}\sqrt{x}$.

$$u(x)=\frac{1}{100}\sqrt{x}\left(1-\frac{\sqrt{\pi}\,x\,\Gamma(7/8)}{400\,\Gamma(11/8)}\right)+\int_0^x \frac{u^2(t)}{\sqrt[8]{x^4-t^4}}\,dt, \qquad 0\le x\le 1.$$

Here we choose $N=21$ points for solving Abel's integral equation.
The absolute, relative, and root mean square errors and the approximate solution of the equation are reported in Table 3, and the error graph is shown in Figure 6. The exact solution and numerical solution of $u(x)$ function is presented in Figure 4.

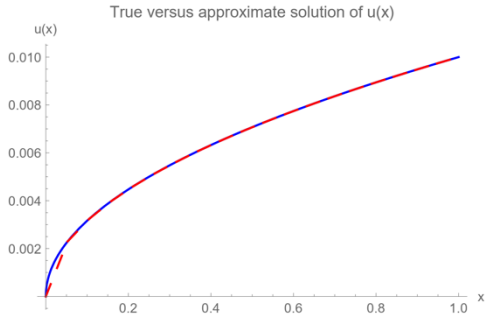


Fig. 4. The exact and numerical solutions curves for example 4.3, the continuous blue line is the exact solution, and the dashed red line is the numerical solution.

Example 4.4: Let us consider the following Abel integral equation,

With the exact solution . $u(x)=\frac{1}{10^{10}}x^2$

$$u(x)=\frac{1}{10^{10}}x^2\left(1-\frac{\sqrt{\pi}}{4}\right)+\frac{1}{\sqrt{\pi}}\int_0^x\frac{u(t)}{\sqrt{x^2-t^2}}dt, \qquad 0\le x\le 1.$$

The absolute, relative, and root mean square errors and the approximate solution of the equation are reported in Table 4, and the error graph is shown in Figure 6. The exact solution and numerical solution of $u(x)$ function is presented in Figure 5.

Table 4. Comparison of error convergence accuracy for example 4.4

Node	True solution $u(x)$	Approximate solution	Absolute error	Relative error	RMS error
0.1	1.E-12	1.27507E-12	2.75072E-13	0.275072	8.29373E-14
0.2	4.E-12	4.36521E-12	3.65211E-13	0.091303	1.10115E-13
0.3	9.E-12	9.41464E-12	4.14638E-13	0.046071	1.25018E-13
0.4	1.6E-11	1.64475E-11	4.47474E-13	0.027967	1.34919E-13
0.5	2.5E-11	2.54714E-11	4.71372E-13	0.018855	1.42124E-13
0.6	3.6E-11	3.64897E-11	4.89677E-13	0.013602	1.47643E-13
0.7	4.9E-11	4.95041E-11	5.04138E-13	0.010289	1.52003E-13
0.8	6.4E-11	6.45158E-11	5.1578E-13	0.008059	1.55513E-13
0.9	8.1E-11	8.15252E-11	5.25247E-13	0.006485	1.58368E-13

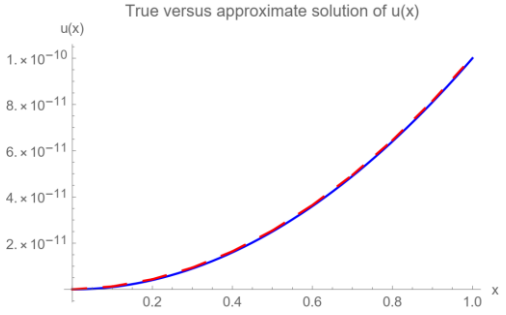
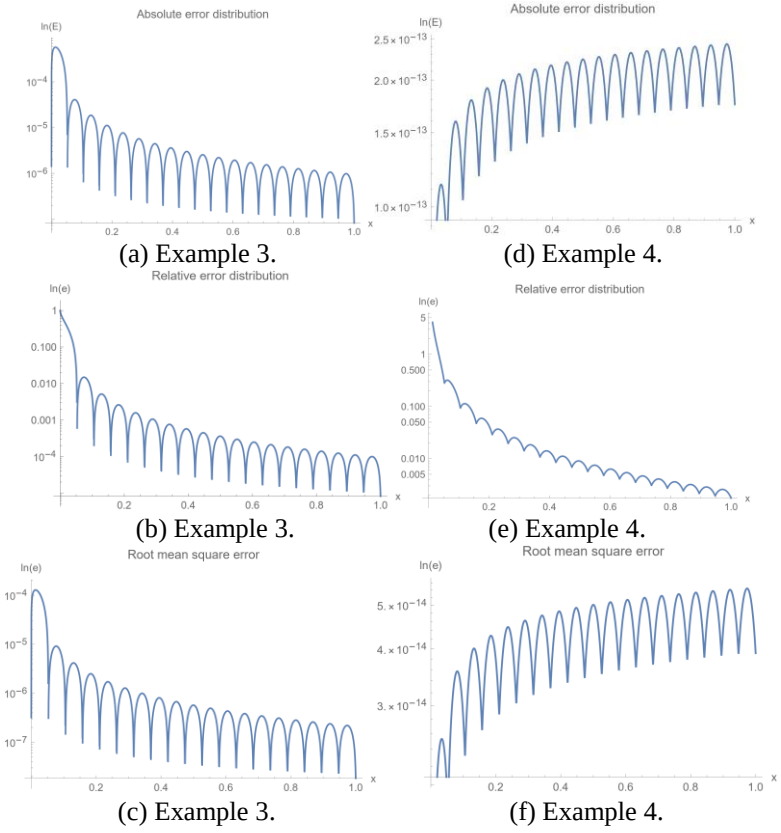


Fig. 5. The exact and numerical solutions curves for example 4.4, the continuous blue line is the exact solution, and the dashed red line is the numerical solution.

Table 5. Comparison of error convergence accuracy for example 4.5

Node	True solution $u(x)$	Approximate solution	Absolute error	Relative error	RMS error
0.1	0.00995	0.00995	3.79888E-6	3.81795E-4	1.14540E-6
0.2	0.03920	0.03918	1.98969E-5	5.07539E-4	5.99914E-6
0.3	0.08598	0.08593	4.97087E-5	5.78141E-4	1.49877E-5
0.4	0.14737	0.14729	8.50952E-5	5.77426E-4	2.56572E-5
0.5	0.21910	0.21929	1.10307E-4	5.02777E-4	3.32589E-5
0.6	0.29712	0.29874	1.62155E-3	5.45753E-3	4.88915E-4
0.7	0.37477	0.37457	2.02619E-4	5.40644E-4	6.10918E-5
0.8	0.44589	0.44946	3.57187E-3	8.01061E-3	1.07696E-3
0.9	0.50350	0.50322	2.79768E-4	5.55642E-4	8.43532E-5



(a) Example 3. (d) Example 4.

(b) Example 3. (e) Example 4.

(c) Example 3. (f) Example 4.

Fig. 6. Figures (a), (b), (c) Error graphs for example 3, Figures (d), (e), (f) Error Graphs for example 4.

Example 4.5: Consider the nonlinear following Abel integral equation with the exact solution $u(x) = x^2 \cos(x)$.

$$u(x)=x^2\cos(x)+\frac{\sqrt{\pi}\,x^{23/10}}{80}\Gamma(4/5)\left(5\left(3-4x^2\right)\text{BesslJ}\left[\frac{23}{10},\,2x\right]\right.$$

$$\left.+6x\,\text{BesslJ}\left[\frac{33}{10},\,2x\right]+\frac{15\,x^{23/10}}{\Gamma(23/10)}\right)-\int_0^x\frac{u^2(t)}{\sqrt[5]{x^2-t^2}}\,dt,\ 0\le x\le 1.$$

Absolute, relative, root mean square errors and approximate solutions of the equation are presented in Table 5. The error plots are shown in Figure 8 and the plots of the exact solution and numerical solution of the $u(x)$ function is presented in Figure 7.

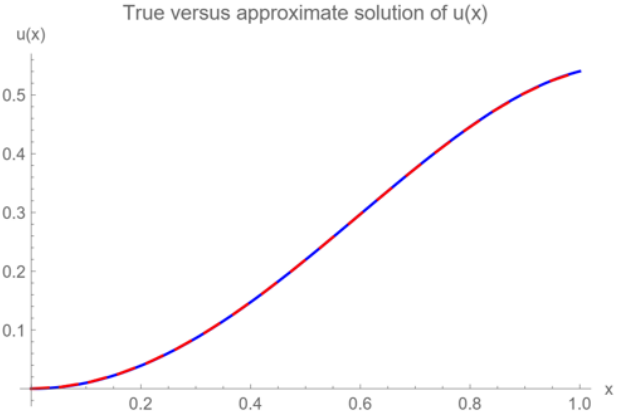
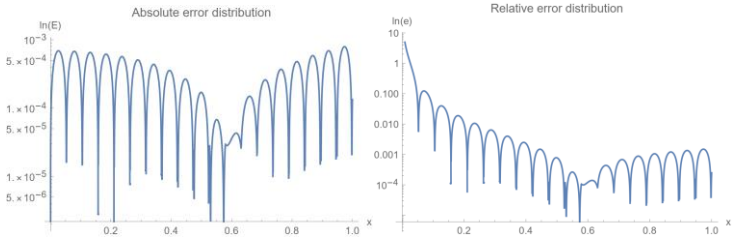
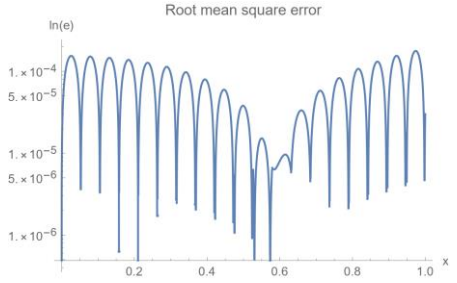


Fig. 7. The exact and numerical solutions curves for example 4.5, the continuous blue line is the exact solution, and the dashed red line is the numerical solution.



(a) Example 5. (b) Example 5.



(c) Example 5.

Fig. 8. Figures (a), (b), (c) Error graphs for example 5.

5. Discussion

In Figure 3, graphs (a), (b), (c) of example 2 and graph (b) of example 1 are convergent. In addition, in graphs (a), and (c) of example 1, the errors are reduced, but they are not convergent. From the comparison of errors and convergence accuracy in Table 1, it can be concluded that the mean square error has been reduced enough and the convergence accuracy of relative errors is better than RMS and absolute error. As you can see in Figure 1, the exact solution converges to the approximate solution. From the study results of Table 2, it can be seen that the convergence accuracy of RMS error is better than relative and absolute error. In Figure 2, Example 1, the comparison of the exact solution and the approximate solution, also shown in Table 1, converges with a very small error. Which shows the correctness of the problems and the correctness of the numerical method. In Figure 6, graphs (a), (b), (c) for example 3, and graph (e) for example 4 are convergent, But figures (d), and (f) of example 4 have sufficiently reduced the error but converge is not because the graph increases upwards. From the results of Table 4, it can be seen that only the absolute error converges and the relative and RMS errors decrease. From Figures 1, 5, and 7, it can be pointed out that the graph of the exact solution and the graph of the approximate solution of the problems are the same, so it shows the correctness of the problems and the accuracy of the method. The results of Table 5 show that its relative error is better and absolute error and RMS error are not convergent. Therefore, in order to achieve the convergence of the subjects and the development of the desired subjects according to the needs of society in the future, we need a revised planning.

6. Conclusion

In this article, the Reproducing Kernel Hilbert Space method is used to solve the integral equation of Abel's type. The absolute error, relative error, root mean square error, accuracy, and convergence of the integral equation are obtained and shown in the tables. The error graph is also compared. To prove the stability of the method, we added a perturbation to the given problem and obtained acceptable numerical results. These numerical experiments show that the proposed method is stable for solving the new problem. In this research, we found that using the Reproducing Kernel Hilbert Space method is very useful and practical for solving integral equations. Because despite the addition of perturbation, the numerical solutions still converge to the exact solution. In the future, we would like to research a new scheme of combining Reproducing Kernel Hilbert Space methods and differential quadrature methods to solve integral equations using fractional derivatives.

References

[1] Atkinson, K. E. (1974). An existence theorem for Abel integral Equations. *SIAM Journal on Mathematical Analysis*, *5*(5), 729- 736.

[2] Alvandi, A., & Paripour, M. (2016). The combined reproducing kernel Method and Taylor series to solve nonlinear Abel's integral Equations with weakly singular kernel. *Cogent Mathematics*, *3*(1), 1250705.

[3] Abdelkawy, M. A., & Ameen, I. G. (2019). A spectral collocation Method for coupled system of two dimensional Abel integral Equations of the second kind. *Information Sciences Letters*, *8*(3), 89-93.

[4] Alvandi, A., & Paripour, M. (2018). Reproducing kernel method for a Class of weakly singular Fredholm integral equations. *Journal of Taibah University for Science*, *12*(4), 409-414.

[5] Abdollahi, N., & Abbasbandy, S. (2022). Reproducing kernel method For Abel's second kind singular integral equations. *Computational Mathematics and Computer Modeling with Applications (CMCMA)*, *1*(2), 27-34.
[6] Cui, M., & Lin, Y. (2009). *Nonlinear numerical analysis in reproducing kernel space*. Nova Science Publishers, Inc.
[7] Farzaneh Javan, S., Abbasbandy, S., & Fariborzi Araghi, M. A. (2017). Application of reproducing kernel Hilbert space method for Solving a class of nonlinear integral equations. *Mathematical Problems in Engineering*, *2017*(1), 7498136.
[8] Mondal, S., & Mandal, B. N. (2018). Application of the Fractional Differential Transform Method to the First Kind Abel Integral Equation. In *Mathematics and Computing: 4th International Conference, ICMC 2018, Varanasi, India, January 9-11, 2018, Revised Selected Papers 4* (pp. 257-267). Springer Singapore.
[9] Kumar, S., Kumar, A., Kumar, D., Singh, J., & Singh, A. (2015). Analytical solution of Abel integral equation arising in Astrophysics via Laplace transform. *Journal of the Egyptian Mathematical Society*, *23*(1), 102-107.
[10] Kumar, K., Pandey, R. K., & Sharma, S. (2019). Approximations of Fractional integrals and Caputo derivatives with application in Solving Abel's integral equations. *Journal of King Saud University-Science*, *31*(4), 692-700.
[11] Mehr, M. (2023). Reproducing kernel Hilbert space method for Solving Abel's integral equations. *Academic research in Educational Sciences*, *4*(11), 147-159.
[12] Pandey, R. K., Sharma, S., & Kumar, K. (2016). Collocation method for generalized Abel's integral equations. *Journal of Computational and Applied Mathematics*, *302*, 118-128.
[13] Singha, N., & Nahak, C. (2019). Solutions of the generalized Abel's Integral equation using laguerre orthogonal Approximation. *Applications and Applied Mathematics: An International Journal (AAM)*, *14*(2), 27.
[14] Shoukralla, E. S., Ahmed, B. M., Sayed, M., & Saeed, A. (2022). Interpolation method for solving Volterra integral equations with weakly singular kernel using an advanced barycentric Lagrange Formula. *Ain Shams Engineering Journal*, *13*(5), 101743.
[15] Thota, S. (2022). Solution of generalized Abel's integral equations by homotopy perturbation method with adaptation in Laplace Transformation. *Sohag J Math*, *9*(2), 29-35.

Authors Profile



Mehrullah Mehr received the M.Sc. and Ph.D. degrees in Applied Mathematics from the Faculty of Science, IKIU University, Iran, in 2012 and 2022, respectively. He is currently an Assistant Professor of Mathematics and Editor-in-Chief of the Scientific-Research Journal of Engineering and the Academic-Research Journal of Baghlan University. His research interests include numerical analysis, theory of differential equations and its application, Stochastic Differential Equations Investigation of Models and numerical Solutions, integral equations, and computational methods for solving differential equations.



Haji Ahmad Joya was born in Dushi Baghlan province. He obtained his master's degree in mathematics – Differential Equations from Al-Farabi Kazakh National University of Kazakhstan in 2015. Since 2018 he has been working as a lecturer and Assistant Professor at Baghlan University. At the same time, he is the head of the Education Department. His research area is advanced mathematics and differential equations.