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A Novel Decision Algorithm for Assessing Digital Economy's Role in Growth: Using Circular q-Rung Orthopair Fuzzy Information Based on WASPAS Method

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Abstract

In real life, decision-making algorithm is pivotal for economic growth and digital economy (EGDE), revolutionizing businesses, governments, and consumer engagement. To unlock its full potential, investing in digital infrastructure and inclusive policies is essential. The goal of this article is to develop a groundbreaking decision-making assessment framework for EGDE, evaluating five alternatives across five key criteria. Leveraging circular q-rung orthopair fuzzy (Cq-ROF) set (Cq-ROFS), our approach has membership degree (MD) and non-membership degree (NMD) while accounting for circular behavior, effectively managing uncertainty in assessments. Building on fuzzy set (FS) theory, our Cq-ROF information encompasses q-rung orthopair fuzzy (q-ROF) set (q-ROFS) as a special case. We define the weighted averaging operator (WAO) and the weighted geometric operator (WGO) within the Cq-ROFS, demonstrating their efficacy through examples and establishing crucial properties. Additionally, we introduce a novel model integrating the weighted aggregated sum product assessment (WASPAS) method with Cq-ROF information and suggest the linguistic terms in this framework. The superiority of our approach is validated through sensitivity analysis and comparative studies, showcasing its potential to tackle uncertainties in EDGE data.

Keywords: Aggregation operators, Circular q-Rung Orthopair fuzzy set, Economic Growth and the Digital Economy, Multi-criteria decision-making approach, WASPAS model.

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1. Introduction

The idea of FS was first introduced by Zadeh [1], where an MD within the range [0,1]. However, traditional FS has limitations in handling complex and uncertain fuzzy information, as it only considers MD without accounting for NMD hesitation. To overcome these restrictions, Atanassov [2] developed the idea of intuitionistic FS (IFS), which incorporates both MD and NMD, providing a more comprehensive framework for dealing with uncertainty. Building upon these foundations, the idea of Pythagorean FS (PyFS) was developed by Yager [3], which further relaxes the constraints on MD and NMD. Moreover, Yager's work on q-ROFS [4] provides a generalized framework that encompasses traditional FS, offering greater flexibility and applicability in modeling complex decision-making problems. The q-ROFS framework allows for a broader range of MD and NMD, enabling more accurate representations of uncertain and ambiguous scenarios.

In the IFS, several notable extensions have been suggested to enhance its applicability. One such extension is the circular IFS (CIFS), introduced by Atanassov [5], which incorporates a radius for each element in the universe of discourse set, representing the uncertainty or vagueness associated with MD and NMD. In CIFS, the sum of MD and NMD is constrained to be ≤ 1, while the radius lies within the interval [0,1]. However, the limited domain of CIFS restricts its ability to handle complex fuzzy information. To address this limitation, the concept of circular PyFS (CPyFS) was introduced by Olgun et al. [6], which expands the domain of CIFS by allowing the sum of the squares of MD and NMD to be ≤ 1. This extension provides greater flexibility in modeling uncertain and ambiguous information. Furthermore, the concept of Cq-ROFS has been explored by Yusoff et al. [7], offering an even more comprehensive environment for dealing with complex fuzzy information. These advancements in FS theory have significant implications for decision-making, pattern recognition and other applications where uncertainty and ambiguity are inherent. For clarity, the advancement of the above FS scenarios is briefly discussed in Figure 1.

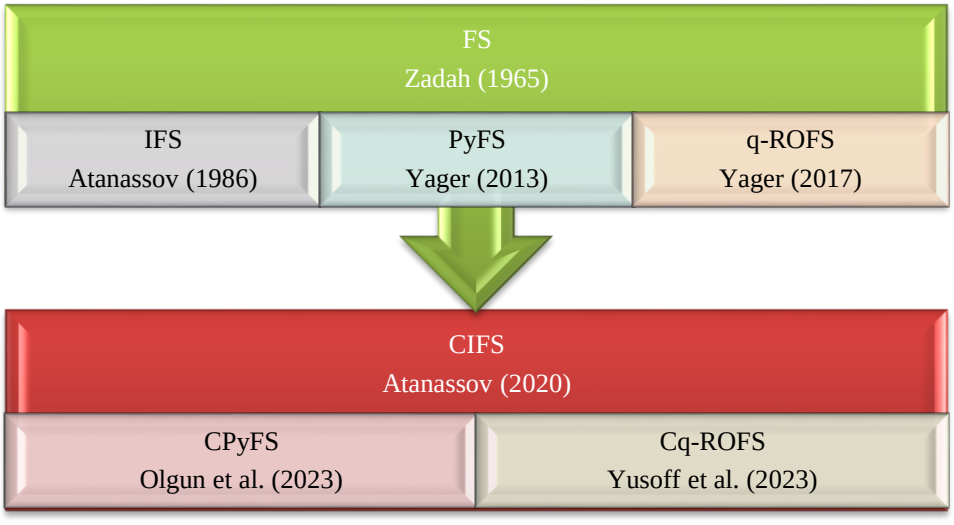


Fig. 1. Advancements in FS theory

1.1.Literature Study

In daily life, MCDM helps decision-makers (Ɗ) navigate situations involving multiple, sometimes conflicting, objectives, leading to more informed and effective choices. It also allows them to weigh trade-offs, evaluate risks, and identify the best possible outcomes. Because of these strengths, MCDM is widely used across various fields such as business, engineering, environmental management, and healthcare for tasks like strategic planning, resource allocation, and policy-making. Hussain et al. [8] initiated Aczel-Alsina AOs in MCDM. MCDM VIKOR method introduced by Wei et al. [9]. Bipolar-valued hesitant FS in MCDM developed by Ullah et al. [10]. The idea of the ELECTRE method in MCDM analysis was adopted by Figueira et al. [11]. An approach toward MCDM and medical diagnosis problems using the concept of spherical FS developed by Mahmood et al. [12].

Economic growth refers to the increase in the inflation-adjusted market value of the goods and services produced by an economy over time. It is often measured by the growth of gross domestic product. The digital economy is the part of the economy that is based on digital computing technologies, encompassing digital communication networks, digital services, e-commerce, and the use of data as an economic asset. The EGDE are key drivers of modern economic growth, transforming how businesses operate, governments function, and consumers engage. Investing in digital infrastructure, skills, and inclusive policies is critical to harness the full potential of this transformation.

The strength of MCDM lies in its ability to handle multiple, often conflicting, criteria and provide a structured approach to evaluating complex decision-making problems. Researchers have explored the application of MCDM in various fields, including fuzzy decision-making, to develop more robust. We may refer to Wang et al. [13], who adopted the TOPSIS method based on prospect theory for interval value hesitant FS MCGDM and its application to a digital economy. Wang [14] introduced the fuzzy method and MCDM to analyze the impact of the digital economy on urban tourism. Silahtaroglu et al. [15] determined the ways to increase economic growth of developing and developed economies: An application with data mining and fuzzy TOPSIS. Ashraf et al. [16] developed the evaluation of economic development policies using a spherical fuzzy extended TODIM model with Z-numbers. Zhang et al. [17] enhance the performance of vocational education in the digital economy with the application of fuzzy logic algorithm.

1.2.Motivation/ Advantages

This research article extends the widely-used WASPAS technique to its Cq-ROF version, building on the foundational work of Zavadskas et al. [18]. The WASPAS technique combines the weighted sum model (WSM) and weighted product model (WPM), offering a comprehensive evaluation approach that surpasses its components. Its popularity stems from its ability to provide effective solutions and consider both qualitative and quantitative factors. The WASPAS technique enables Ɗ to evaluate and rank alternatives based on their preferences and the relative importance of each criterion. By integrating Cq-ROFS, we leverage the strengths of FS theory to handle uncertainty in linguistic terms. This article demonstrates the application of the Cq-ROF

WASPAS technique in the context of EGDE. A detailed overview of the WASPAS technique frameworks is provided in Table 1.

Table 1. Previous publications of the WASPAS model

Year	References	Technique	Approach	Experimental Result
2013	Zavadskas et al. [19]	WASPAS and MULTIMOORA	MCDM	Robustness
2014	Bitarafan et al. [20]	SWARA-WASPAS	MCDM	Health monitoring
2015	Kazimieras et al. [21]	WASPAS	MCDM	Construction
2016	Ghorabaee et al. [22]	WASPAS	MCDM	Green suppliers
2017	Ghorabaee et al. [23]	CRITIC-WASPAS	MCGDM	Third-party logistics
2018	Alam et al. [24]	AHP-WASPAS	MCGDM	Computing service
2019	Alinzhad et al. [25]	WASPAS	MCGDM	Illustrative example
2020	Dorfeshan et al. [26]	MABAC and WASPAS	MCGDM	Aircraft maintenance
2021	Simic et al. [27]	WASPAS	MCDM	Last-mile delivery
2022	Jabbarova et al. [28]	WASPAS	MCGDM	Plat-form selection
2023	Fang et al. [29]	WASPAS	MCDM	Illustrative example
2024	Tufail et al. [30]	WASPAS	MCDM	Agricultural diagnosis
2025	Proposed work	WASPAS	MCDM	Digital economy

In real-world scenarios, criteria often exhibit interdependencies, influencing each other in complex ways. For example, decisions on EGDE can yield enhanced benefits and quality. To capture these relationships and improve decision-making accuracy, researchers are incorporating interdependence between criteria. However, existing WASPAS techniques, as shown in Table 1, fall short in handling Cq-ROF information. To address this limitation, we extend the WASPAS method to accommodate Cq-ROF information. This article elaborates on the evaluation process and algorithm for the MCDM approach, rendering existing methods a special case of our proposed WASPAS model. The proposed aggregation operators (AOs) and WASPAS technique demonstrate superiority due to their ability to handle complex interdependencies and uncertainty.

The WASPAS technique's strength lies in its ability to handle problems with both benefit and non-benefit criteria, where alternatives and criteria are interconnected. For instance, decisions on EGDE can have a ripple effect, enhancing benefits and quality. To capture these complex relationships, researchers prioritize interdependence between criteria. However, existing works in Table 1 are limited in handling Cq-ROF information. Our proposed WASPAS technique overcomes this limitation by incorporating Cq-ROF information based on linguistic terms and prioritized weights for criteria importance. Key advantages include handling circular information and considering the radius of MD and NMD. The proposed technique subsumes existing methods in Table 1 as special cases, demonstrating its effectiveness and potential for future applications.

1.3. Objectives/Contribution

The main objectives and contributions are:

1. Introducing Cq-ROFWAO and Cq-ROFGAO operators in the Cq-ROFS environment, with illustrative examples.
2. The mathematical properties of the proposed operators are used to ensure their validity and applicability in decision-making scenarios.

3. Adopting linguistic terms in the Cq-ROF context for alternative evaluation and criteria importance assessment.
4. Develop a solution for prioritizing weights for criteria importance, ensuring the weights sum to 1, and apply it to the MCDM approach.
5. Developing a new WASPAS technique model based on initiated AOs and utilizing it to aggregate the decision matrix.
6. Resolve an experimental result using the MCDM approach in the Cq-ROF context to demonstrate the reliability of the proposed theory.
7. Assess five alternatives for EGDE development using the MCDM approach.
8. Investigate the influence of different parametric values on the ranking of alternatives.
9. Compare the proposed approach with existing works to evaluate its reliability and effectiveness.

Figure 2 showcases the geometric representation of our research findings.

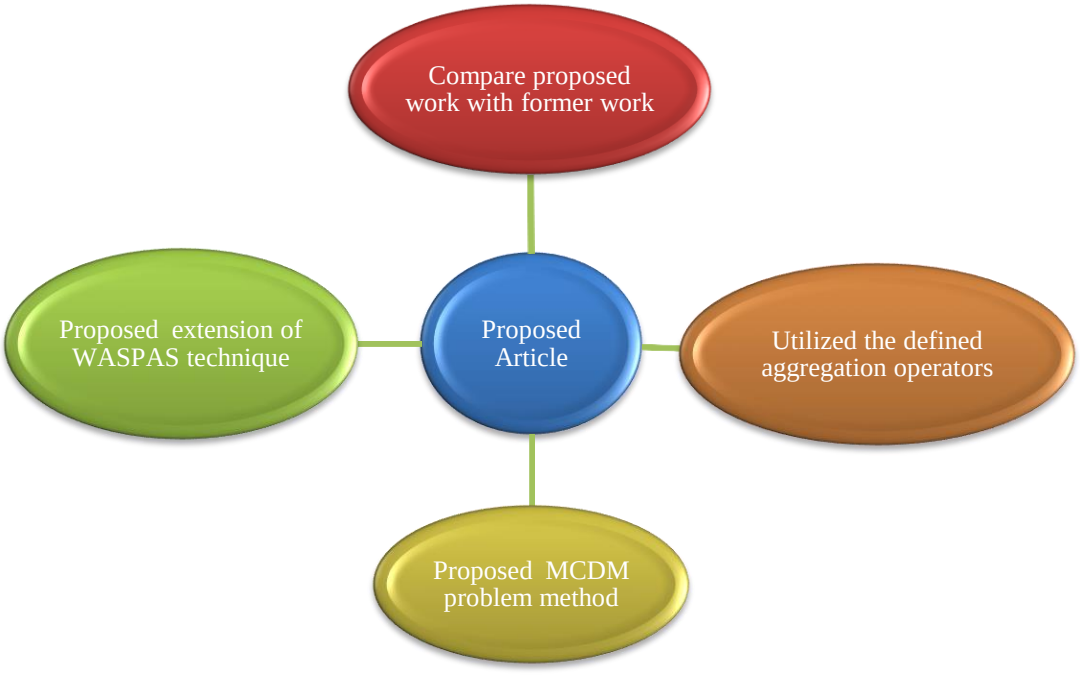


Fig. 2. A brief overview of the proposed theoretical perspective

1.4.Article Structure

This article presents a Cq-ROF framework combining AOs and the WASPAS method. The structure is as follows:

- Introduction (Section 1)
- WASPAS technique overview (Section 2)
- Cq-ROF fundamentals (Section 3)
- Novel AOs (Section 4)

- WASPAS extension in Cq-ROF framework (Section 5)
- Case study (Section 6)
- Sensitivity analysis (Section 7)
- Comparative study (Section 8)
- Conclusion and future directions (Section 9)

2. Algorithm of WASPAS Technique

The WASPAS technique, introduced by Zavadskas et al. [18], is a robust decision-making approach. The WASPAS method combines the benefits of WSM and WPM to provide a comprehensive evaluation framework. Below, we outline the step-by-step procedure of the WASPAS technique.

Step 1: Construct the informative DM and integrate two kinds of criteria, such as benefit criteria and non-benefit criteria.

$$M_{\mathfrak{m}\times\mathfrak{n}} = \begin{matrix} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \vdots \\ \mathbb{A}_{\mathfrak{m}} \end{matrix} \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 & \dots & \mathbb{C}_{\mathfrak{n}} \\ \mathcal{X}_{11} & \mathcal{X}_{12} & \dots & \mathcal{X}_{1\mathfrak{n}} \\ \mathcal{X}_{21} & \mathcal{X}_{22} & \dots & \mathcal{X}_{2\mathfrak{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{X}_{\mathfrak{m}1} & \mathcal{X}_{\mathfrak{m}2} & \dots & \mathcal{X}_{\mathfrak{m}\mathfrak{n}} \end{bmatrix}$$

Step 2: Normalized the DM for each entry according to its performance, such as:

$$\widetilde{M}_{\mathfrak{m}\times\mathfrak{n}} = \begin{cases} \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}} = \dfrac{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}{max_{\mathfrak{i}}\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}; \text{ for benefit criteria} \\ \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}} = \dfrac{min_{\mathfrak{i}}\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}; \text{ for non} - \text{beneficial criteria} \end{cases}$$

Step 3: Accumulate WSM and WPM by using the following expression:

$$WSM = \sum_{\mathfrak{j}=1}^{\mathfrak{n}} \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}\, \mathfrak{w}_{\mathfrak{j}}$$

$$WPM = \prod_{\mathfrak{j}=1}^{\mathfrak{n}} \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}^{\mathfrak{w}_{\mathfrak{j}}}$$

Step 4: The joint criteria of WSM and WPM illustrate the total importance of alternatives, called the WASPAS method. In the following convex formula, we integrate the values of individual WASPAS methods:

$$WASPAS = \lambda \sum_{\mathfrak{j}=1}^{\mathfrak{n}} \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}\, \mathfrak{w}_{\mathfrak{j}} + (1-\lambda) \prod_{\mathfrak{j}=1}^{\mathfrak{n}} \widetilde{\mathcal{X}_{\mathfrak{i}\mathfrak{j}}}^{\mathfrak{w}_{\mathfrak{j}}}, \qquad \lambda \in [0,1]$$

Step 5: After finding score values and ranking by joint criteria, we demonstrate the reliable alternative.

3. Preliminaries

In this section, we review related operators, definitions, and operations needed in this article.

3.1.Weighted Averaging and Weighted Geometric Operators

In this subsection, we will recall and analyze some powerful AOs, which are the q-ROF WAO (q-ROFWAO) and q-ROF WGO (q-ROFWGO), respectively.

Definition 1 [31]**:** The q-ROFWAO for q-ROFVs $\beta_J(J = 1,2,3, \dots, ŋ)$ is of the form:

$$q - ROFW\beta O(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \sum_{J=1}^{ŋ} ϖ_J \beta_J$$

Where the weighted vector $ϖ = (ϖ_1, ϖ_2, \dots, ϖ_J)$ such that $J(J = 1,2,3, \dots, ŋ)$ satisfied the constraint $ϖ_J \geq 0$ and $\sum ϖ_J = 1$. The collection of q-ROFVs of A_J aggregate by using the q-ROFWAO, we get again q-ROFV as:

$$q - ROFW\beta O(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\sqrt[q]{1 - \prod_{J=1}^{ŋ} (1 - \mathbb{M}^q{}_J)^{ϖ_J}}, \prod_{J=1}^{ŋ} \mathbb{N}_J{}^{ϖ_J} \right)$$

Definition 2 [31]**:** The q-ROFWGO for q-ROFVs $\beta_J(J = 1,2,3, \dots, ŋ)$ is of the form:

$$q - ROFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \prod_{J=1}^{ŋ} \beta_J{}^{ϖ_J}$$

Where the weighted vector $ϖ = (ϖ_1, ϖ_2, \dots, ϖ_J)$ such that $J(J = 1,2,3, \dots, ŋ)$ satisfied the constraint $ϖ_J \geq 0$ and $\sum ϖ_J = 1$. The collection of q-ROFVs of A_J aggregate by using the q-ROFWGO, we get again q-ROFV as:

$$q - ROFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\prod_{J=1}^{ŋ} \mathbb{M}_J{}^{ϖ_J}, \sqrt[q]{1 - \prod_{J=1}^{ŋ} (1 - \mathbb{N}^q{}_J)^{ϖ_J}} \right)$$

3.2.Circular q-Rung Orthopair Fuzzy Information

This subsection provides Cq-ROFS, basic operational laws, score values, and accuracy values. Cq-ROFS is a valuable tool for addressing unpredictable and uncertain information in real-world problems, which gives us more effective decisions.

Definition 3 [7]**:** A Cq-ROFS β on the universe of discourse set Ụ is defined as: $\beta = \left\{ \left(\mathbb{M}_\beta(\mathcal{X}), \mathbb{N}_\beta(\mathcal{X}); \mathbb{R}_\beta(\mathcal{X}) \right) | \mathcal{X} \in Ụ \right\}$, where $\mathbb{M}_\beta(\mathcal{X})$ and $\mathbb{N}_\beta(\mathcal{X})$ represent the MD and NMD, respectively. These grades satisfy the constraint $0 \leq \left(\mathbb{M}_\beta(\mathcal{X}) \right)^q + \left(\mathbb{N}_\beta(\mathcal{X}) \right)^q \leq 1$ for some $q \in \mathbb{Z}^+$, while $\mathbb{R}_\beta(\mathcal{X})$ denotes the radius of the circular uncertainty region surrounding the MD and NMD of $\mathcal{X}$. Additionally, the refusal grade is given by $ħ_\beta(\mathcal{X}) = \sqrt[q]{1 - \left(\mathbb{M}_\beta(\mathcal{X}) \right)^q - \left(\mathbb{N}_\beta(\mathcal{X}) \right)^q}$. For simplicity, of Cq-ROF value (Cq-ROFV) is denoted by $\beta_J = (\mathbb{M}_{\beta_J}, \mathbb{N}_{\beta_J})$, $(J = 1,2, \dots, ŋ)$.

Definition 4 [7]**:** Let $\beta_1 = \left(\mathbb{M}_{\beta_1}, \mathbb{N}_{\beta_1}, \mathbb{R}_{\beta_1}\right)$ and $\beta_2 = \left(\mathbb{M}_{\beta_2}, \mathbb{N}_{\beta_2}, \mathbb{R}_{\beta_2}\right)$ be the two Cq-ROFVs, for some $q \in \mathbb{Z}^+$with different radius and $\mathbb{K} > 0$. Then,

$$\beta_1 \oplus \beta_2 = \left(\sqrt[q]{\left(\mathbb{M}_{\beta_1}\right)^q + \left(\mathbb{M}_{\beta_2}\right)^q - \left(\mathbb{M}_{\beta_1}\right)^q \left(\mathbb{M}_{\beta_2}\right)^q}, \mathbb{N}_{\beta_1}\mathbb{N}_{\beta_2}; \mathbb{R}_{\beta_1}\mathbb{R}_{\beta_2}\right) \tag{3.1}$$

$$\beta_1 \otimes \beta_2 = \left(\mathbb{M}_{\beta_1}\mathbb{M}_{\beta_2}, \sqrt[q]{\left(\mathbb{N}_{\beta_1}\right)^q + \left(\mathbb{N}_{\beta_2}\right)^q - \left(\mathbb{N}_{\beta_1}\right)^q \left(\mathbb{N}_{\beta_2}\right)^q}; \mathbb{R}_{\beta_1}\mathbb{R}_{\beta_2}\right) \tag{3.2}$$

$$\mathbb{K}\beta_1 = \left(\sqrt[q]{1 - \left(1 - \left(\mathbb{M}_{\beta_1}\right)^q\right)^{\mathbb{K}}}, \left(\mathbb{N}_{\beta_1}\right)^{\mathbb{K}}; \left(\mathbb{R}_{\beta_1}\right)^{\mathbb{K}}\right) \tag{3.3}$$

$$(\beta_1)^{\mathbb{K}} = \left(\left(\mathbb{M}_{\beta_1}\right)^{\mathbb{K}}, \sqrt[q]{1 - \left(1 - \left(\mathbb{N}_{\beta_1}\right)^q\right)^{\mathbb{K}}}; \left(\mathbb{R}_{\beta_1}\right)^{\mathbb{K}}\right) \tag{3.4}$$

$$(\beta_1)^c = \left\{\mathbb{N}_{\beta_1}, \mathbb{M}_{\beta_1}, \mathbb{R}_{\beta_1}\right\} \tag{3.5}$$

Definition 5 [32]**:** The score function $\mathcal{S}$ and accuracy function $\mathcal{A}$ of a Cq-ROFV $\beta = \left(\mathbb{M}_\beta, \mathbb{N}_\beta, \mathbb{R}_\beta\right)$ is defined by

$$\mathcal{S}(\beta) = \left(\left(\mathbb{M}_\beta\right)^q - \left(\mathbb{N}_\beta\right)^q\right) \times \mathbb{R}_\beta \in [-1,1] \tag{3.6}$$

$$\mathcal{A}(\beta) = \left(\left(\mathbb{M}_\beta\right)^q + \left(\mathbb{N}_\beta\right)^q\right) \times \mathbb{R}_\beta \in [0,1] \tag{3.7}$$

The order relation for comparing two Cq-ROFVs can be specified as:

- $\beta_1 < \beta_2$ if and only if $\mathcal{S}(\beta_1) < \mathcal{S}(\beta_2)$ or $\mathcal{S}(\beta_1) = \mathcal{S}(\beta_2)$
- $\beta_1 < \beta_2$ if and only if $\mathcal{A}(\beta_1) < \mathcal{A}(\beta_2)$ or $\mathcal{A}(\beta_1) = \mathcal{A}(\beta_2)$

4. Some Aggregation Operators

This section presents the development of the WAO and WGO in the framework of Cq-ROFS. By introducing a radius for each element, the Cq-ROFS model offers a more refined way to represent uncertainty, making it especially useful in decision-making scenarios. The effectiveness of this approach is illustrated through several practical examples.

4.1.Circular q-Rung Orthopair Fuzzy Weighted Averaging Operator

In this part of the article, we introduce the Cq-ROF WAO (Cq-ROFWAO) and demonstrate its practical application with an experimental example. We also explore several important properties of the Cq-ROFWAO within the Cq-ROFS framework, emphasizing both its theoretical foundation and practical value.

Definition 6: The Cq-ROFWAO for Cq-ROFVs of $\beta_J (J = 1,2,3, \dots, ŋ)$ is of the form:

$$Cq - ROFW\beta O(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \sum_{J=1}^{ŋ} \mathfrak{w}_J \beta_J \tag{4.1}$$

Where the weighted vector $\mathfrak{w} = (\mathfrak{w}_1, \mathfrak{w}_2, \dots, \mathfrak{w}_J)$ such that for all $\mathfrak{w}_J \geq 0$ and satisfied the condition $\sum \mathfrak{w}_J = 1$.

Theorem 1: Let A_J be the collection of Cq-ROFVs, by using the Cq-ROFWAO, the aggregated value is again a Cq-ROFV and is given below:

$$Cq - ROFW\beta O(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\sqrt[q]{1 - \prod_{J=1}^{ŋ} (1 - \mathbb{M}^q{}_J)^{\mathfrak{w}_J}}, \prod_{J=1}^{ŋ} \mathbb{N}_J{}^{\mathfrak{w}_J}, \prod_{J=1}^{ŋ} \mathbb{R}_J{}^{\mathfrak{w}_J} \right) \tag{4.2}$$

Proof: We are required to check the result by utilizing the mathematical induction method, and it is shown as follows:

For $ŋ = 2$, based on the operational law of Cq-ROFV, we get

$\mathfrak{w}_1\beta_1 = \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_1)^{\mathfrak{w}_1}}, \mathbb{N}_1{}^{\mathfrak{w}_1}, \mathbb{R}_1{}^{\mathfrak{w}_1}\right)$, $\mathfrak{w}_2\beta_2 = \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_2)^{\mathfrak{w}_2}}, \mathbb{N}_2{}^{\mathfrak{w}_2}, \mathbb{R}_2{}^{\mathfrak{w}_2}\right)$

$\mathfrak{w}_1\beta_1 \oplus \mathfrak{w}_2\beta_2$

$= \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_1)^{\mathfrak{w}_1}}, \mathbb{N}_1{}^{\mathfrak{w}_1}, \mathbb{R}_1{}^{\mathfrak{w}_1}\right) \oplus \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_2)^{\mathfrak{w}_2}}, \mathbb{N}_2{}^{\mathfrak{w}_2}, \mathbb{R}_2{}^{\mathfrak{w}_2}\right)$

$$= \left(\sqrt[q]{\left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_1)^{\mathfrak{w}_1}}\right)^q + \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_2)^{\mathfrak{w}_2}}\right)^q - \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_1)^{\mathfrak{w}_1}}\right)^q \left(\sqrt[q]{1 -} \right.} \right.$$
$$\left. \left. \mathbb{N}_1{}^{\mathfrak{w}_1}\mathbb{N}_2{}^{\mathfrak{w}_2}, \mathbb{R}_1{}^{\mathfrak{w}_1}\mathbb{R}_2{}^{\mathfrak{w}_2} \right)$$

$$= \left(\sqrt[q]{1 - (1 - \mathbb{M}^q{}_1)^{\mathfrak{w}_1}(1 - \mathbb{M}^q{}_2)^{\mathfrak{w}_2}}, \mathbb{N}_1{}^{\mathfrak{w}_1}\mathbb{N}_2{}^{\mathfrak{w}_2}, \mathbb{R}_1{}^{\mathfrak{w}_1}\mathbb{R}_2{}^{\mathfrak{w}_2}\right)$$

$$= \left(\sqrt[q]{1 - \prod_{J=1}^{2} (1 - \mathbb{M}^q{}_J)^{\mathfrak{w}_J}}, \prod_{J=1}^{2} \mathbb{N}_J{}^{\mathfrak{w}_J}, \prod_{J=1}^{2} \mathbb{R}_J{}^{\mathfrak{w}_J} \right)$$

Assume that the result holds for $ŋ = l$ that is

$$Cq - ROFW\beta O(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\sqrt[q]{1 - \prod_{J=1}^{l} (1 - \mathbb{M}^q{}_J)^{\mathfrak{w}_J}}, \prod_{J=1}^{l} \mathbb{N}_J{}^{\mathfrak{w}_J}, \prod_{J=1}^{l} \mathbb{R}_J{}^{\mathfrak{w}_J} \right)$$

To prove this result for $ŋ = l + 1$, based on the operational law of Cq-ROFV, we get

$$Cq-ROFW\beta O\left(\beta_1,\beta_2,\beta_3,\dots,\beta_{\mathrm{J}},\beta_{\mathrm{J}+1}\right)=\sum_{\mathrm{J}=1}^{l+1}\varpi_{\mathrm{J}}\beta_{\mathrm{J}}=\sum_{\mathrm{J}=1}^{l}\varpi_{\mathrm{J}}\beta_{\mathrm{J}}\oplus\varpi_{\mathrm{J}+1}\beta_{\mathrm{J}+1}$$

$$=\begin{pmatrix}\left(\sqrt[q]{1-\prod_{\mathrm{J}=1}^{l}\left(1-\mathbb{M}^{q}{}_{\mathrm{J}}\right)^{\varpi_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{l}\mathbb{N}_{\mathrm{J}}{}^{\varpi_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l}\mathbb{R}_{\mathrm{J}}{}^{\varpi_{\mathrm{J}}}\right)\oplus\\ \left(\sqrt[q]{1-\left(1-\mathbb{M}^{q}{}_{l+1}\right)^{\varpi_{l+1}}},\mathbb{N}_{l+1}{}^{\varpi_{l+1}},\mathbb{R}_{l+1}{}^{\varpi_{l+1}}\right)\end{pmatrix}$$

Proceed as we did for $\eta=2$ that is

$$=\left(\sqrt[q]{1-\prod_{\mathrm{J}=1}^{l+1}\left(1-\mathbb{M}^{q}{}_{\mathrm{J}}\right)^{\varpi_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{l+1}\mathbb{N}_{\mathrm{J}}{}^{\varpi_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l+1}\mathbb{R}_{\mathrm{J}}{}^{\varpi_{\mathrm{J}}}\right)$$

Hence, the result holds for $\eta=l+1$.

Example 1: The Cq-ROFVs of the universe of discourse set $\ddot{\mathrm{U}}=\{\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3\}$ are $\mathcal{X}_1=(0.90,0.70,0.80)$, $\mathcal{X}_2=(0.95,0.55,0.75)$, and $\mathcal{X}_3=(0.92,0.62,0.77)$ with the corresponding weighted vector of criteria is $\varpi=(0.25,0.40,0.35)$, $\sum_{\mathrm{J}=1}^{3}\varpi_{\mathrm{J}}=1$. Then, find $Cq-ROFW\beta O(\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3)$, whenever $q=4$.

Solution: $Cq-ROFW\beta O(\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3)=$

$$\begin{pmatrix}\sqrt[4]{1-\left(\left(1-0.90^4\right)^{0.25}\right)\left(\left(1-0.95^4\right)^{0.40}\right)\left(\left(1-0.92^4\right)^{0.35}\right)},\\ (0.70)^{0.25}(0.55)^{0.40}(0.62)^{0.35},\\ (0.80)^{0.25}(0.75)^{0.40}(0.77)^{0.35}\end{pmatrix}$$

$$=(0.9303,0.6092,0.7693)$$

Thus, it is evident that our proposed operator, as defined in Eq. (**4. 2**), yields a valid Cq-ROFVs. This outcome demonstrates the effectiveness and reliability of our proposed approach, showcasing its ability to handle Cq-ROF information in a consistent and meaningful manner.

4.2.Properties of Circular q-Rung Orthopair Fuzzy Weighted Averaging Operator

Some reliable properties of the initiated operator Cq-ROFWAO are outlined below:

Idempotency: The collection of a finite number of Cq-ROFVs, $\mathcal{X}_{\mathrm{J}}=\left(\mathbb{M}_{\mathcal{X}_{\mathrm{J}}},\mathbb{N}_{\mathcal{X}_{\mathrm{J}}},\mathbb{R}_{\mathcal{X}_{\mathrm{J}}}\right)$ for all $\mathrm{J}=1,2,\dots,\eta$. Then, the idempotency property of the Cq-ROFWAO is:

$$\mathrm{Cq-ROFWAO}\left(\mathcal{X}_1,\mathcal{X}_2,\dots,\mathcal{X}_{\eta}\right)=\mathcal{X}$$

Boundedness: The collection of a finite number of Cq-ROFVs, $\mathcal{X}_J = (\mathbb{M}_{\mathcal{X}J}, \mathbb{N}_{\mathcal{X}J}, \mathbb{R}_{\mathcal{X}J})$ for all $J = 1,2,\dots,ŋ$. If $\mathcal{X}^- = \left(\min_J\left(\mathbb{M}_{\mathcal{X}_J}\right), \max_J\left(\mathbb{N}_{\mathcal{X}_J}\right), \min_J\left(\mathbb{R}_{\mathcal{X}_J}\right)\right)$ and $\mathcal{X}^+ = \left(\max_J\left(\mathbb{M}_{\mathcal{X}_J}\right), \min_J\left(\mathbb{N}_{\mathcal{X}_J}\right), \min_J\left(\mathbb{R}_{\mathcal{X}_J}\right)\right)$. Then, the boundedness property of the Cq-ROFWAO is:

$$\mathcal{X}^- \leq \mathrm{Cq-ROFWAO}(\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_ŋ) \leq \mathcal{X}^+$$

Monotonicity: Assume that the two collections of a finite number of Cq-ROFVs, $\mathcal{X}_J = (\mathbb{M}_{\mathcal{X}J}, \mathbb{N}_{\mathcal{X}J}, \mathbb{R}_{\mathcal{X}J})$ and $\mathcal{X}^*{}_J = (\mathbb{M}^*{}_{\mathcal{X}J}, \mathbb{N}^*{}_{\mathcal{X}J}, \mathbb{R}^*{}_{\mathcal{X}J})$ for all $J = 1,2,\dots,ŋ$. If $\mathcal{X}_J \leq \mathcal{X}^*{}_J$ such that $\sqrt[q]{\mathbb{M}_{\mathcal{X}J}{}^q} < \sqrt[q]{\mathbb{M}^*{}_{\mathcal{X}J}{}^q}$, $\sqrt[q]{\mathbb{N}_{\mathcal{X}J}{}^q} > \sqrt[q]{\mathbb{N}^*{}_{\mathcal{X}J}{}^q}$, and $\sqrt[q]{\mathbb{R}_{\mathcal{X}J}{}^q} < \sqrt[q]{\mathbb{R}^*{}_{\mathcal{X}J}{}^q}$ for all $J = 1,2,\dots,ŋ$. Then, the monotonicity property of the Cq-ROFWAO is:

$$\mathrm{Cq-ROFWAO}(\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_ŋ) \leq \mathrm{Cq-ROFWAO}(\mathcal{X}_1{}^*, \mathcal{X}_2{}^*, \dots, \mathcal{X}_ŋ{}^*)$$

4.3.Circular q-Rung Orthopair Fuzzy Weighted Geometric Operator

In this subsection, we introduce a Cq-ROF WGO (Cq-ROFWGO) and demonstrate its application through an experimental result. Additionally, we investigate and propose several desirable properties of the Cq-ROFWGO within the Cq-ROF framework, highlighting its theoretical and practical significance.

Definition 7: The Cq-ROFWGO for Cq-ROFVs of $\beta_J (J = 1,2,3,\dots,ŋ)$ is of the form:

$$Cq - ROFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \prod_{J=1}^{ŋ} \beta_J{}^{w_J} \tag{4.3}$$

Where the weighted vector $w = (w_1, w_2, \dots, w_J)$ such that for all $w_J \geq 0$ and satisfied the condition $\sum w_J = 1$.

Theorem 2: Let A_J be the collection of Cq-ROFVs, by using the Cq-ROFWGO, the aggregated value is again a Cq-ROFV and is given below:

$$Cq - ROFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\prod_{J=1}^{ŋ} \mathbb{M}_J{}^{w_J}, \sqrt[q]{1 - \prod_{J=1}^{ŋ}(1 - \mathbb{N}^q{}_J)^{w_J}}, \prod_{J=1}^{ŋ} \mathbb{R}_J{}^{w_J}\right) \tag{4.4}$$

Proof: We are required to check the result by utilizing the mathematical induction method, and it is shown as follows:

For $ŋ = 2$, based on the operational law of Cq-ROFV, we get

$$w_1\beta_1 = \left(\mathbb{M}_1{}^{w_1}, \sqrt[q]{1 - (1 - \mathbb{N}^q{}_1)^{w_1}}, \mathbb{R}_1{}^{w_1}\right), w_2\beta_2 = \left(\mathbb{M}_2{}^{w_2}, \sqrt[q]{1 - (1 - \mathbb{N}^q{}_2)^{w_2}}, \mathbb{R}_2{}^{w_2}\right)$$

$$w_1\beta_1 \otimes w_2\beta_2 = \left(\mathbb{M}_1{}^{w_1}, \sqrt[q]{1 - (1 - \mathbb{N}^q{}_1)^{w_1}}, \mathbb{R}_1{}^{w_1}\right) \otimes \left(\mathbb{M}_2{}^{w_2}, \sqrt[q]{1 - (1 - \mathbb{N}^q{}_2)^{w_2}}, \mathbb{R}_2{}^{w_2}\right)$$

$$= \left(\begin{matrix} \mathbb{M}_1{}^{\mathfrak{w}_1}\mathbb{M}_2{}^{\mathfrak{w}_2}, \\ \sqrt[q]{\left(\sqrt[q]{1-(1-\mathbb{N}^q{}_1)^{\mathfrak{w}_1}}\right)^q + \left(\sqrt[q]{1-(1-\mathbb{N}^q{}_2)^{\mathfrak{w}_2}}\right)^q - \left(\sqrt[q]{1-(1-\mathbb{N}^q{}_1)^{\mathfrak{w}_1}}\right)^q \left(\sqrt[q]{1-(1-\mathbb{N}^q{}_2)^{\mathfrak{w}_2}}\right)^q}, \\ \mathbb{R}_1{}^{\mathfrak{w}_1}\mathbb{R}_2{}^{\mathfrak{w}_2} \end{matrix} \right)$$

$$= \left(\mathbb{M}_1{}^{\mathfrak{w}_1}\mathbb{M}_2{}^{\mathfrak{w}_2}, \sqrt[q]{1-(1-\mathbb{N}^q{}_1)^{\mathfrak{w}_1}(1-\mathbb{N}^q{}_2)^{\mathfrak{w}_2}}, \mathbb{R}_1{}^{\mathfrak{w}_1}\mathbb{R}_2{}^{\mathfrak{w}_2} \right)$$

$$= \left(\prod_{\mathrm{J}=1}^{2} \mathbb{M}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}}, \sqrt[q]{1-\prod_{\mathrm{J}=1}^{2}\left(1-\mathbb{N}^q{}_{\mathrm{J}}\right)^{\mathfrak{w}_{\mathrm{J}}}}, \prod_{\mathrm{J}=1}^{2} \mathbb{R}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} \right)$$

Assume that the result holds for $\mathfrak{y} = l$ that is

$$Cq - ROFWGO\left(\beta_1, \beta_2, \beta_3, \dots, \beta_{\mathrm{J}}\right) = \left(\prod_{\mathrm{J}=1}^{l} \mathbb{M}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}}, \sqrt[q]{1-\prod_{\mathrm{J}=1}^{l}\left(1-\mathbb{N}^q{}_{\mathrm{J}}\right)^{\mathfrak{w}_{\mathrm{J}}}}, \prod_{\mathrm{J}=1}^{l} \mathbb{R}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} \right)$$

To prove this result for $\mathfrak{y} = l + 1$, based on the operational law of Cq-ROFV, we get

$$Cq - ROFWGO\left(\beta_1, \beta_2, \beta_3, \dots, \beta_{\mathrm{J}}, \beta_{\mathrm{J}+1}\right) = \prod_{\mathrm{J}=1}^{l+1} \beta_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} = \prod_{\mathrm{J}=1}^{l} \beta_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} \otimes \beta_{\mathrm{J}+1}{}^{\mathfrak{w}_{\mathrm{J}+1}}$$

$$= \left(\left(\prod_{\mathrm{J}=1}^{l} \mathbb{M}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}}, \sqrt[q]{1-\prod_{\mathrm{J}=1}^{l}\left(1-\mathbb{N}^q{}_{\mathrm{J}}\right)^{\mathfrak{w}_{\mathrm{J}}}}, \prod_{\mathrm{J}=1}^{l} \mathbb{R}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} \right) \otimes \left(\mathbb{M}_{l+1}{}^{\mathfrak{w}_{l+1}}, \sqrt[q]{1-(1-\mathbb{N}^q{}_{l+1})^{\mathfrak{w}_{l+1}}}, \mathbb{R}_{l+1}{}^{\mathfrak{w}_{l+1}} \right) \right)$$

Proceed as we did for $\mathfrak{y} = 2$ that is

$$= \left(\prod_{\mathrm{J}=1}^{l+1} \mathbb{M}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}}, \sqrt[q]{1-\prod_{\mathrm{J}=1}^{l+1}\left(1-\mathbb{N}^q{}_{\mathrm{J}}\right)^{\mathfrak{w}_{\mathrm{J}}}}, \prod_{\mathrm{J}=1}^{l+1} \mathbb{R}_{\mathrm{J}}{}^{\mathfrak{w}_{\mathrm{J}}} \right)$$

Hence, the result holds for $\mathfrak{y} = l + 1$.

Example 2: The Cq-ROFVs of the universe of discourse set $\ddot{\mathrm{U}} = \{\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3\}$ are $\mathcal{X}_1 = (0.90,0.70,0.80)$, $\mathcal{X}_2 = (0.95,0.55,0.75)$, and $\mathcal{X}_3 = (0.92,0.62,0.77)$ with the corresponding weighted vector of criteria is $\mathfrak{w} = (0.25,0.40,0.35)$, $\sum_{\mathrm{J}=1}^{3} \mathfrak{w}_{\mathrm{J}} = 1$. Then, find $Cq - ROFWGO(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3)$, whenever $q = 4$.

Solution: $Cq - ROFWGO(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3) =$

$$\left(\begin{array}{c} (0.90)^{0.25}(0.95)^{0.40}(0.92)^{0.35}, \\ \sqrt[4]{1-\left(\left(1-0.70^4\right)^{0.25}\right)\left(\left(1-0.55^4\right)^{0.40}\right)\left(\left(1-0.62^4\right)^{0.35}\right)}, \\ (0.80)^{0.25}(0.75)^{0.40}(0.77)^{0.35} \end{array} \right)$$

$$= (0.9268, 0.6227, 0.7693)$$

Thus, it is evident that our proposed operator, as defined in Eq. (**4.4**), yields a valid Cq-ROFVs. This outcome demonstrates the effectiveness and reliability of our proposed approach, showcasing its ability to handle Cq-ROF information in a consistent and meaningful manner.

4.4.Properties of Circular q-Rung Orthopair Fuzzy Weighted Geometric Operator

Some reliable properties of the initiated operator Cq-ROFWGO are outlined below:

Idempotency: The collection of a finite number of Cq-ROFVs, $\mathcal{X}_\mathrm{J} = \left(\mathbb{M}_{\mathcal{X}_\mathrm{J}}, \mathbb{N}_{\mathcal{X}_\mathrm{J}}, \mathbb{R}_{\mathcal{X}_\mathrm{J}}\right)$ for all $\mathrm{J} = 1,2,\ldots,\mathrm{n}$. Then, the idempotency property of the Cq-ROFWGO is:

$$\mathrm{Cq-ROFWGO}(\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_\mathrm{n}) = \mathcal{X}$$

Boundedness: The collection of a finite number of Cq-ROFVs, $\mathcal{X}_\mathrm{J} = \left(\mathbb{M}_{\mathcal{X}_\mathrm{J}}, \mathbb{N}_{\mathcal{X}_\mathrm{J}}, \mathbb{R}_{\mathcal{X}_\mathrm{J}}\right)$ for all $\mathrm{J} = 1,2,\ldots,\mathrm{n}$. If $\mathcal{X}^- = \left(\min_\mathrm{J}\left(\mathbb{M}_{\mathcal{X}_\mathrm{J}}\right), \max_\mathrm{J}\left(\mathbb{N}_{\mathcal{X}_\mathrm{J}}\right), \min_\mathrm{J}\left(\mathbb{R}_{\mathcal{X}_\mathrm{J}}\right)\right)$ and $\mathcal{X}^+ = \left(\max_\mathrm{J}\left(\mathbb{M}_{\mathcal{X}_\mathrm{J}}\right), \min_\mathrm{J}\left(\mathbb{N}_{\mathcal{X}_\mathrm{J}}\right), \min_\mathrm{J}\left(\mathbb{R}_{\mathcal{X}_\mathrm{J}}\right)\right)$. Then, the boundedness property of the Cq-ROFWGO is:

$$\mathcal{X}^- \leq \mathrm{Cq-ROFWGO}(\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_\mathrm{n}) \leq \mathcal{X}^+$$

Monotonicity: Assume that the two collections of a finite number of Cq-ROFVs, $\mathcal{X}_\mathrm{J} = \left(\mathbb{M}_{\mathcal{X}_\mathrm{J}}, \mathbb{N}_{\mathcal{X}_\mathrm{J}}, \mathbb{R}_{\mathcal{X}_\mathrm{J}}\right)$ and $\mathcal{X}^*_\mathrm{J} = \left(\mathbb{M}^*_{\mathcal{X}_\mathrm{J}}, \mathbb{N}^*_{\mathcal{X}_\mathrm{J}}, \mathbb{R}^*_{\mathcal{X}_\mathrm{J}}\right)$ for all $\mathrm{J} = 1,2,\ldots,\mathrm{n}$. If $\mathcal{X}_\mathrm{J} \leq \mathcal{X}^*_\mathrm{J}$ such that $\sqrt[q]{\mathbb{M}_{\mathcal{X}_\mathrm{J}}{}^q} < \sqrt[q]{\mathbb{M}^*_{\mathcal{X}_\mathrm{J}}{}^q}$, $\sqrt[q]{\mathbb{N}_{\mathcal{X}_\mathrm{J}}{}^q} > \sqrt[q]{\mathbb{N}^*_{\mathcal{X}_\mathrm{J}}{}^q}$, and $\sqrt[q]{\mathbb{R}_{\mathcal{X}_\mathrm{J}}{}^q} < \sqrt[q]{\mathbb{R}^*_{\mathcal{X}_\mathrm{J}}{}^q}$ for all $\mathrm{J} = 1,2,\ldots,\mathrm{n}$. Then, the monotonicity property of the Cq-ROFWGO is:

$$\mathrm{Cq-ROFWGO}(\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_\mathrm{n}) \leq \mathrm{Cq-ROFWGO}(\mathcal{X}_1{}^*, \mathcal{X}_2{}^*, \ldots, \mathcal{X}_\mathrm{n}{}^*)$$

5. Algorithm of WASPAS Model in Circular q-Rung Orthopair Fuzzy framework

This section focuses on the WASPAS model, which integrates weighted sum and weighted product approaches in the Cq-ROF environment. The WASPAS method is denoted by $\mathcal{Q}_\mathrm{i}$ combines two well-known approaches: WSM, denoted by $\mathcal{Q}_\mathrm{i}^S$, this approach calculates the weighted sum of the criteria values. WPM denoted by $\mathcal{Q}_\mathrm{i}^P$, this approach calculates the weighted product of the criteria values. The WASPAS technique proves especially effective in handling uncertain data within assessment processes, allowing Ɗ to articulate their preferences using MD, NMD, and radius values

within the closed interval [0,1]. Unlike the PyFS model, where the squared sum of MD and NMD can exceed 1, the Cq-ROFS imposes a stricter constraint: the sum of the $q \geq 2$ of MD and NMD must not exceed 1. To streamline the evaluation process, we employ linguistic terms to assess alternatives against each criterion, as illustrated in Table 2**.** Additionally, Table 3 provides linguistic terms for the importance weights of the criteria.

Table 2. For the evaluation of alternatives, the linguistic terms were utilized

Linguistic term	**MD**	**NMD**	**Radius**
Superior (SUP)	0.95	0.05	0.50
Dominant (DOM)	0.90	0.15	0.53
Optimal (OPT	0.85	0.25	0.55
Extensive (EXT)	0.80	0.35	0.58
Considerable (CON)	0.75	0.45	0.60
Moderate (MOD)	0.70	0.55	0.63
Nominal (NOM)	0.65	0.65	0.65
Minimal (MIN)	0.60	0.75	0.68
Negligible (NEG)	0.55	0.80	0.68

Table 3. For the evaluation of the weight of the criteria, the linguistic terms

Linguistic term	**MD**	**NMD**	**Radius**
Strongly agree (SA)	0.95	0.25	0.60
Agree (AG)	0.80	0.35	0.58
Neutral (NE)	0.65	0.45	0.55
Disagree (DA)	0.50	0.55	0.53
Strongly disagree (SD)	0.35	0.65	0.50

Step 1: The Ɗ will use the linguistic terms in Table 2 to assess alternatives against the criteria, considering both beneficial and non-beneficial criteria types in the DM.

$$M_{\mathrm{m}\times\mathrm{n}} = \begin{matrix} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \vdots \\ \mathbb{A}_\mathrm{m} \end{matrix} \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 & \dots & \mathbb{C}_\mathrm{n} \\ \mathcal{X}_{11} & \mathcal{X}_{12} & \dots & \mathcal{X}_{1\mathrm{n}} \\ \mathcal{X}_{21} & \mathcal{X}_{22} & \dots & \mathcal{X}_{2\mathrm{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{X}_{\mathrm{m}1} & \mathcal{X}_{\mathrm{m}2} & \dots & \mathcal{X}_{\mathrm{mn}} \end{bmatrix} \tag{5.1}$$

Step 2: When DM includes both beneficial and non-beneficial criteria, normalization is necessary to ensure consistency. To normalize the DM, the following equations can be applied:

$$\widetilde{\mathcal{X}}_{\mathrm{ij}} = \begin{cases} \left(\mathbb{M}_{\mathrm{ij}}, \mathbb{N}_{\mathrm{ij}}, \mathbb{R}_{\mathrm{ij}}\right); & \text{for benefit criteria} \\ \left(\mathbb{N}_{\mathrm{ij}}, \mathbb{M}_{\mathrm{ij}}, \mathbb{R}_{\mathrm{ij}}\right); & \text{for non} - \text{beneficial criteria} \end{cases} \tag{5.2}$$

$$\tag{5.3}$$

After normalization, the original DM presented in Eq. (5.1) is transformed into a normalized DM presented in Eq. (5.4), enabling further analysis and decision-making.

$$\widetilde{M}_{\mathrm{m}\times\mathrm{n}} = \begin{matrix}\mathbb{A}_1\\ \mathbb{A}_2\\ \vdots\\ \mathbb{A}_\mathrm{m}\end{matrix}\begin{bmatrix}\mathbb{C}_1 & \mathbb{C}_2 & \dots & \mathbb{C}_\mathrm{n}\\ \widetilde{\mathcal{X}}_{11} & \widetilde{\mathcal{X}}_{12} & \dots & \widetilde{\mathcal{X}}_{1\mathrm{n}}\\ \widetilde{\mathcal{X}}_{21} & \widetilde{\mathcal{X}}_{22} & \dots & \widetilde{\mathcal{X}}_{1\mathrm{n}}\\ \vdots & \vdots & \ddots & \vdots\\ \widetilde{\mathcal{X}}_{\mathrm{m}1} & \widetilde{\mathcal{X}}_{\mathrm{m}2} & \dots & \widetilde{\mathcal{X}}_{\mathrm{mn}}\end{bmatrix} \tag{5.4}$$

Step 3: Determining the prioritized weighted vector for criteria importance is crucial, as criteria weights can vary. The Ɗ uses linguistic terms from Table 3 to evaluate criteria importance, then applies Eq. (5.5) for defuzzification and Eq. (5.6) for normalization to obtain the weight vector.

$$\mathfrak{w}_\mathrm{J}^{\mathcal{S}} = \left(1 - \left(\mathbb{M}_\mathrm{J}\right)^q - \left(\mathbb{N}_\mathrm{J}\right)^q\right) \times \mathbb{R}_\mathrm{J} \tag{5.5}$$

$$\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}} = \frac{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}{\sum_{\mathrm{J}=1}^{\mathrm{n}} \mathfrak{w}_\mathrm{J}^{\mathcal{S}}} \tag{5.6}$$

The Ɗ can choose from various defuzzification techniques for Cq-ROFVs, selecting the one that best fits their needs. The weighted vector $\widetilde{\mathfrak{w}^{\mathcal{S}}} = \left(\widetilde{\mathfrak{w}_1^{\mathcal{S}}}, \widetilde{\mathfrak{w}_2^{\mathcal{S}}}, \dots, \widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}\right)$ is prioritized and satisfies the condition $\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}} \geq 0$ such that $\sum \widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}} = 1$.

Step 4: The WASPAS method's first suggestion $\mathcal{Q}_\mathrm{I}^{S}$ in the framework of Cq-ROFS, it is defined in Eq. (5.7).

$$\mathcal{Q}_\mathrm{I}^{S} = Cq - ROFW\beta = \sum_{\mathrm{J}=1}^{\mathrm{n}} \widetilde{\mathcal{X}_{\mathrm{IJ}}}\, \widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}} \tag{5.7}$$

Given that Eq. (5.7) involves multiplication and addition operations, these operations are executed according to Eq. (4.2) and presented in Eq. (5.8).

$$\begin{aligned}\mathcal{Q}_\mathrm{I}^{S} = Cq - ROFW\beta &= \sum_{\mathrm{J}=1}^{\mathrm{n}} \widetilde{\mathcal{X}_{\mathrm{IJ}}}\, \widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}\\ &= \left(\sqrt[q]{1 - \prod_{\mathrm{J}=1}^{\mathrm{n}} \left(1 - \mathbb{M}_{\widetilde{\mathcal{X}_{\mathrm{IJ}}}}^{q}\right)^{\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}}}, \prod_{\mathrm{J}=1}^{\mathrm{n}} \mathbb{N}_{\widetilde{\mathcal{X}_{\mathrm{IJ}}}}^{\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}}, \prod_{\mathrm{J}=1}^{\mathrm{n}} \mathbb{R}_{\widetilde{\mathcal{X}_{\mathrm{IJ}}}}^{\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}}\right)\end{aligned} \tag{5.8}$$

The WASPAS method's first suggestion $\mathcal{Q}_\mathrm{I}^{P}$ in the framework of Cq-ROFS is defined in Eq. (5.9).

$$\mathcal{Q}_\mathrm{I}^{P} = Cq - ROFWG = \prod_{\mathrm{J}=1}^{\mathrm{n}} \widetilde{\mathcal{X}_{\mathrm{IJ}}}^{\widetilde{\mathfrak{w}_\mathrm{J}^{\mathcal{S}}}} \tag{5.9}$$

Given that Eq. (5.9) involves multiplication and addition operations, these operations are executed according to Eq. (4.4) and presented in Eq. (5.10).

$$\mathcal{Q}_{\ddot{\imath}}^{P} = Cq - ROFWG = \prod_{J=1}^{\eta} \widetilde{\mathcal{X}}_{\ddot{\imath}J}^{\widetilde{w_J^S}} \tag{5.10}$$

$$= \left(\prod_{J=1}^{\eta} \mathbb{M}_{\widetilde{\mathcal{X}}_{\ddot{\imath}J}}^{\widetilde{w_J^S}}, \sqrt[q]{1 - \prod_{J=1}^{\eta} \left(1 - \mathbb{N}_{\widetilde{\mathcal{X}}_{\ddot{\imath}J}}^{q}\right)^{\widetilde{w^S}}}, \prod_{J=1}^{\eta} \mathbb{R}_{\widetilde{\mathcal{X}}_{\ddot{\imath}J}}^{\widetilde{w_J^S}} \right)$$

We aggregate the normalized DM Eq. (5.4) using the prioritized weighted vector and Eq. (5.8) and Eq. (5.10) to calculate $\mathcal{Q}_{\ddot{\imath}}^{S}$ and $\mathcal{Q}_{\ddot{\imath}}^{P}$ respectively. The overall assessment is then obtained using the WASPAS convex formula, $\mathcal{Q}_{\ddot{\imath}}$, as presented in Eq. (5.11).

$$\mathcal{Q}_{\ddot{\imath}} = WASPAS = \lambda \mathcal{Q}_{\ddot{\imath}}^{S} + (1 - \lambda)\mathcal{Q}_{\ddot{\imath}}^{P} \tag{5.11}$$

Given that Eq. (5.11) involves scalar multiplication in both terms, we apply Eq. (3.3) to calculate each term and then use the addition operation from Eq. (3.1) to combine the results.

Step 5: Determined the score values for each alternative of Eq. (5.8), Eq. (5.10), and Eq. (5.11) with the help of Eq. (3.6). In cases of tied score values, we use the accuracy function Eq. (3.7) to distinguish between alternatives. The alternatives are then ranked, and the best option is determined.

To illustrate the process, a flowchart of the WASPAS-based MCDM approach is provided in Figure 3, enhancing understanding and convenience.

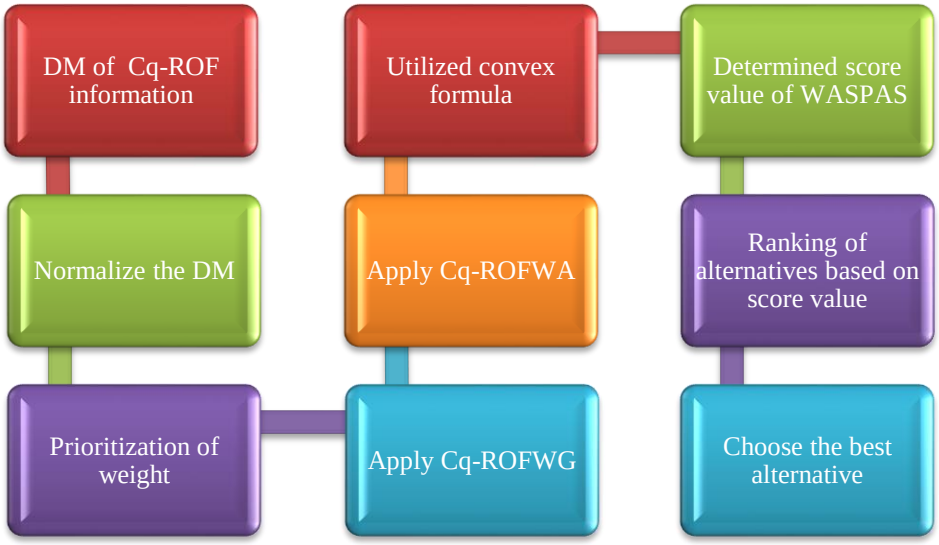


Fig. 3. WASPAS decision-making procedure

6. Case Study

This case study applies the initiated WASPAS model within the Cq-ROF framework to address challenges in EGDE, utilizing an MCDM approach.

6.1.Problem Statement

Economic growth is characterized by a sustained increase in the inflation-adjusted market value of goods and services produced within an economy over a specified period. This growth is typically measured by the percentage change in gross domestic product, which serves as a comprehensive indicator of economic activity. On the other hand, the digital economy represents a significant and rapidly evolving segment of the overall economy, driven by digital computing technologies, digital communication networks, and the increasing importance of data as a valuable economic asset. The digital economy encompasses a broad range of activities, including e-commerce, digital services, online platforms, and the use of digital technologies to enhance business operations and create new opportunities. To explore the complex relationships between EGDE, we will consider a set of five distinct alternatives, which are outlined below:

- **Digital Transformation and Economic Development** ($\mathbb{A}_1$)**:** The process by which economies evolve through the integration of digital technologies such as artificial intelligence across all sectors, driving changes in productivity, job creation, infrastructure, and societal well-being.
- **The Impact of the Digital Economy on Economic Growth** ($\mathbb{A}_2$)**:** An examination of how the digital economy, comprised of e-commerce, digital finance, platforms, and data-driven services, contributes to or influences the overall growth in gross domestic product, productivity, and innovation.
- **Technological Innovation and Sustainable Economic Growth** ($\mathbb{A}_3$)**:** The relationship between the development and application of new technologies and their ability to support long-term, environmentally sustainable, and inclusive economic progress.
- **Harnessing Digital Technologies for Economic Progress** ($\mathbb{A}_4$)**:** The strategic use of digital tools and innovations (e.g., artificial intelligence, blockchain, mobile apps) to enhance business efficiency, governance, trade, and labor market outcomes, driving economic improvement.
- **The Role of Digitalization in Modern Economic Systems** ($\mathbb{A}_5$)**:** Explores how the increasing digitization of services, production, communication, and logistics alters the functioning and structure of contemporary economies.

When assessing various types of EGDE, several key criteria come into play. These factors are crucial in determining which type of EGDE strategy is most suitable for a particular context or organization. Some of the most important criteria are outlined below:

- **Gross Domestic Product Growth** ($\mathbb{C}_1$)**:** Gross domestic product growth measures the increase in the value of goods and services produced in a country over time. In the context of the digital economy, it includes the output from digital sectors such as information and communication technology, digital services, e-commerce, and online platforms.
- **Digital Infrastructure and Connectivity** ($\mathbb{C}_2$)**:** Digital infrastructure refers to the physical and organizational structures needed for digital operations, including internet networks, data centers, and mobile connectivity. Connectivity

measures the population's access to reliable, fast, and affordable internet and mobile services.

- **Human Capital and Digital Skills** ($\mathbb{C}_3$)**:** Human capital encompasses the education, skills, and abilities of a population. Digital skills refer to the knowledge and capabilities required to use digital tools and technologies effectively, such as programming, data analysis, and online communication.
- **Innovation and Technology Adoption** ($\mathbb{C}_4$)**:** Innovation involves creating new products, services, or processes that improve efficiency or value. Technology adoption is the extent to which businesses, governments, and individuals use new digital technologies like artificial intelligence in their daily activities.
- **Regulatory Environment and Digital Governance** ($\mathbb{C}_5$)**:** The regulatory environment includes the laws, policies, and frameworks that govern economic and digital activities. Digital governance refers to how governments manage digital services, data privacy, cyber security, and online rights to create a safe and efficient digital economy.

This manuscript introduces an innovative approach to tackling the complexities and challenges linked to EGDE. By developing a new framework based on the Cq-ROFS, we aim to improve the accuracy and reliability of decision-making in these areas. The proposed AOs offer clear advantages over existing methods, enabling more effective integration of EGDE indicators. One of the main strengths of our approach is its ability to handle the inherent uncertainty and complexity in this domain, resulting in more robust and accurate evaluations. Additionally, the integration of the WASPAS technique further strengthens the framework by allowing it to process diverse and heterogeneous data, ultimately enhancing the precision of decision outcomes.

6.2.Experimental Result

The proposed WASPAS extension within the Cq-ROF framework Eq. $(\mathbf{5.8}, \mathbf{5.10}, \mathbf{5.11})$ can be effectively applied to EGDE selection, addressing the aforementioned challenges. This approach leverages FS information to aggregate data and provide more accurate estimations in real-world scenarios. Consider that the above-mentioned alternative has five criteria. Assumed that in this case study, the data is hypothetical and the decision values provided by the $\mathcal{D}$ are in the form of the Cq-ROF framework. The weighted vector $\widetilde{\mathfrak{w}^{\mathcal{S}}} = \left(\widetilde{\mathfrak{w}_1^{\mathcal{S}}}, \widetilde{\mathfrak{w}_2^{\mathcal{S}}}, \ldots, \widetilde{\mathfrak{w}_\jmath^{\mathcal{S}}}\right)$ is prioritized and satisfies the condition $\widetilde{\mathfrak{w}_\jmath^{\mathcal{S}}} \geq \mathbf{0}$ such that $\sum \widetilde{\mathfrak{w}_\jmath^{\mathcal{S}}} = \mathbf{1}$.

Step 1: The $\mathcal{D}$ will use the linguistic terms in Table 2 to assess alternatives against the criteria, considering both beneficial and non-beneficial criteria types are stored in Table 4 (for $q = 3$).

Table 4. DM of each alternative for EGDE

$\mathcal{D}$	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathbb{A}_1$	MIN	OPT	NOM	EXT	NEG
$\mathbb{A}_2$	OPT	DOM	SUP	OPT	NEG
$\mathbb{A}_3$	SUP	CON	MOD	SUP	NOM
$\mathbb{A}_4$	OPT	SUP	MIN	DOM	CON
$\mathbb{A}_5$	DOM	OPT	NEG	DOM	OPT

Step 2: The $\mathfrak{D}$ takes the first, second, and fourth criteria as benefits, while the third and fifth criteria are non-beneficial. The normalized DM are showcased in Table 5, by utilizing Eq. (5.2) and Eq. (5.3), respectively.

Table 5. Normalized DM of each alternative for EGDE

$\mathfrak{D}$	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathbb{A}_1$	(0.60,0.75,0.68)	(0.85,0.25,0.55)	(0.65,0.65,0.65)	(0.80,0.35,0.58)	(0.80,0.55,0.68)
$\mathbb{A}_2$	(0.85,0.25,0.55)	(0.90,0.15,0.53)	(0.05,0.95,0.50)	(0.85,0.25,0.55)	(0.80,0.55,0.68)
$\mathbb{A}_3$	(0.95,0.05,0.50)	(0.75,0.45,0.60)	(0.55,0.70,0.63)	(0.95,0.05,0.50)	(0.65,0.65,0.65)
$\mathbb{A}_4$	(0.85,0.25,0.55)	(0.95,0.05,0.50)	(0.75,0.60,0.68)	(0.90,0.15,0.53)	(0.45,0.75,0.60)
$\mathbb{A}_5$	(0.90,0.15,0.53)	(0.85,0.25,0.55)	(0.80,0.55,0.68)	(0.90,0.15,0.53)	(0.25,0.85,0.55)

Step 3: To determine the prioritized weighted vector for criteria importance, the $\mathfrak{D}$ uses the linguistic terms from Table 3 (for q = 3), as shown in the second column of Table 6. The importance weights are listed in the third column. These weights are then defuzzified by utilizing Eq. (5.5) and normalized by using Eq. (5.6), with the results presented in the fourth and fifth columns of Table 6, respectively.

Table 6. Prioritization process of weight for the importance of the criteria

Criteria	**Weight for criteria**	**Linguistic terms**	**Defuzzified weight**	**Normalized weight**
$\mathbb{C}_1$	SA	(0.95,0.25,0.60)	0.08	0.06
$\mathbb{C}_2$	DA	(0.50,0.55,0.53)	0.38	0.27
$\mathbb{C}_3$	SD	(0.35,0.65,0.50)	0.34	0.24
$\mathbb{C}_4$	AG	(0.80,0.35,0.58)	0.26	0.18
$\mathbb{C}_5$	NE	(0.65,0.45,0.55)	0.35	0.25

Step 4: The normalized DM presented in Table 5 is aggregated using the proposed AOs Eq. (5.8) and Eq. (5.10) and utilizes the prioritized weights presented in Table 6. The accumulated result of $\mathcal{Q}_{\mathbb{i}}^{S}$ and $\mathcal{Q}_{\mathbb{i}}^{P}$ are showcased in Table 7.

Table 7. Aggregated DM of WSM and WPM

Alternatives	**Cq-ROFWA=$\mathcal{Q}_{\mathbb{i}}^{S}$**	**Cq-ROFWG=$\mathcal{Q}_{\mathbb{i}}^{P}$**
$\mathbb{A}_1$	(0.78,0.43,0.62)	(0.76,0.54,0.62)
$\mathbb{A}_2$	(0.81,0.37,0.56)	(0.43,0.74,0.56)
$\mathbb{A}_3$	(0.81,0.32,0.59)	(0.71,0.57,0.59)
$\mathbb{A}_4$	(0.85,0.24,0.57)	(0.73,0.56,0.57)
$\mathbb{A}_5$	(0.81,0.36,0.57)	(0.63,0.63,0.57)

Using the values from Table 7 and applying Eq. (3.3), $\lambda\mathcal{Q}_{\mathbb{i}}^{S}$ and $(1-\lambda)\mathcal{Q}_{\mathbb{i}}^{P}$ are calculated for $\lambda = 0.5$, as shown in Table 8. The overall score of $\mathcal{Q}_{\mathbb{i}}$, is obtained by using Eq. (3.1), and the results are presented in Table 8.

Table 8. Final result of the WASPAS method

Alternatives	λQ_i^S	$(1-\lambda)Q_i^P$	$Q_i = \lambda Q_i^S + (1-\lambda)Q_i^P$
$\mathbb{A}_1$	(0.65,0.66,0.79)	(0.63,0.73,0.79)	(0.77,0.48,0.62)
$\mathbb{A}_2$	(0.68,0.60,0.75)	(0.34,0.86,0.75)	(0.70,0.52,0.56)
$\mathbb{A}_3$	(0.68,0.57,0.77)	(0.58,0.76,0.77)	(0.76,0.43,0.59)
$\mathbb{A}_4$	(0.73,0.49,0.76)	(0.60,0.75,0.76)	(0.80,0.37,0.57)
$\mathbb{A}_5$	(0.68,0.60,0.76)	(0.51,0.79,0.76)	(0.74,0.48,0.57)

Step 5: The score values of Table 8 and Table 7 are obtained by using Eq. (3.6), and the ranking of each alternative is stored in Table 9.

Table 9. Score values and ranking of initiated work

Alternatives	Score of Q_i^S	Ranking of Q_i^S	Score of Q_i^P	Ranking of Q_i^P	Score of Q_i	Ranking of Q_i
$\mathbb{A}_1$	0.25	5	0.18	1	0.39	3
$\mathbb{A}_2$	0.27	4	-0.18	5	0.27	5
$\mathbb{A}_3$	0.29	2	0.10	3	0.40	2
$\mathbb{A}_4$	0.35	1	0.12	2	0.49	1
$\mathbb{A}_5$	0.27	3	0.00	4	0.34	4

Notably, $\mathbb{A}_4$ (harnessing digital technologies for economic progress) is the best alternative for both Cq-ROFWA and WASPAS, while $\mathbb{A}_1$ (digital transformation and economic development) is the best for Cq-ROFWG. The ranking differences highlight the importance of the D choice of AOs. Figure 4 visually illustrates the behavior and ranking of alternatives.

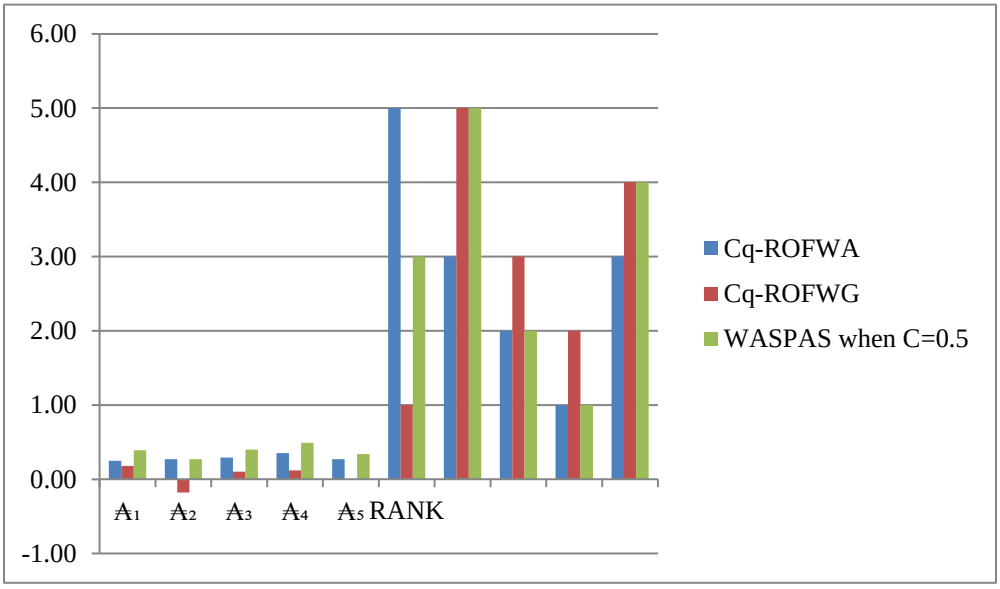


Fig. 4. Geometrical representation derived from the initiated work

Figure 4 illustrates the score values and rankings of alternatives derived from the Cq-ROFWA and Cq-ROFWG operators, along with the WASPAS technique under the Cq-

ROF environment. The results show $\mathbb{A}_4$ as the most suitable alternative for Cq-ROFWA and WASPAS, while $\mathbb{A}_1$ ranks highest for Cq-ROFWG. Notably, the green line in Figure 4, representing the WASPAS technique, highlights the optimal option.

7. Sensitivity Analysis

This section investigates the impact of the proposed operators, Eq. (5.8), Eq. (5.10), and Eq. (5.11), by applying them to the data in Table 5 along with a prioritized weighted vector. We evaluate how varying the parameter $\boldsymbol{\lambda}$ influences the ranking of alternatives in the context of EGDE. The results reveal a consistent ranking pattern for parameter values ranging from 0.1 to 0.9, as detailed in Table 10, underscoring the robustness of the proposed approach.

Table 10. Sensitivity score values and ranking

$\boldsymbol{\lambda}$	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$	$\mathbb{A}_4$	$\mathbb{A}_5$	**Ranking**
$\boldsymbol{\lambda_1 = 0.1}$	0.35	-0.04	0.28	0.34	0.16	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_2 = 0.2}$	0.36	0.05	0.32	0.39	0.21	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_3 = 0.3}$	0.37	0.13	0.35	0.43	0.26	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_4 = 0.4}$	0.38	0.20	0.37	0.46	0.30	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_5 = 0.5}$	0.39	0.27	0.40	0.49	0.34	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_6 = 0.6}$	0.40	0.32	0.42	0.52	0.38	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$
$\boldsymbol{\lambda_7 = 0.7}$	0.41	0.38	0.44	0.55	0.41	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_1 > \mathbb{A}_2$
$\boldsymbol{\lambda_8 = 0.8}$	0.42	0.42	0.47	0.57	0.44	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_1$
$\boldsymbol{\lambda_9 = 0.9}$	0.42	0.47	0.48	0.59	0.47	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_1$

Table 10 demonstrates that $\mathbb{A}_4$ consistently emerges as the best alternative across different parametric values, except when $\lambda = 0.1$. This sensitivity analysis reinforces the robustness of $\mathbb{A}_4$ as the most suitable option for fostering economic growth and advancing the digital economy, highlighting its stability under varying conditions.

8. Comparative Analysis

To validate the proposed theory and assess its compatibility, we conduct a comparative analysis using the data from Table 5. This comparative analysis highlights the effectiveness and feasibility of our suggested approaches. By applying the initiated work to the given data, we obtain ranking results that are then compared with earlier techniques to evaluate the proposed techniques' supremacy and validity. Liu et al. [31] introduced some q-ROF AOs. Khan et al. [33] initiated q-ROF Aczel Alsina power AOs. Ali et al. [34] initiated the generalization of q-ROF Aczel Alsina AOs. Farid et al. [35] exposed generalized q-ROF Einstein interactive geometric AOs. Peng et al. [36] evaluated exponential operation and AOs for q-ROFS. Du et al. [37] introduced q-rung orthopair triangular fuzzy linguistic Frank AOs. Jana et al. [38] derived some Dombi aggregation of q-ROF numbers. Liu et al. [39] developed q-ROF Heronian mean operators. Using the data from Table 5, the comparison results are summarized in Table 11.

Table 11. Demonstration of the comparative performance of the proposed approach and the existing approach

References/Technique	Output/Ranking	Out come
Liu et al. [31]	Evaluation of the data in Table 5 is not feasible	Failed
Khan et al. [33]	Evaluation of the data in Table 5 is not feasible	Failed
Ali et al. [34]	Evaluation of the data in Table 5 is not feasible	Failed
Farid et al. [35]	Evaluation of the data in Table 5 is not feasible	Failed
Peng et al. [36]	Evaluation of the data in Table 5 is not feasible	Failed
Du et al. [37]	Evaluation of the data in Table 5 is not feasible	Failed
Jana et al. [38]	Evaluation of the data in Table 5 is not feasible	Failed
Liu et al. [39]	Evaluation of the data in Table 5 is not feasible	Failed
Cq-ROFWA	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_1$	$\mathbb{A}_4$
Cq-ROFWG	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2$	$\mathbb{A}_1$
WASPAS	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5 > \mathbb{A}_2$	$\mathbb{A}_4$

As observed in Table 11, the proposed method demonstrates high feasibility and effectiveness due to its enhanced features. In contrast, existing techniques are unable to solve the Table 5 data, primarily due to their limited capabilities. The main limitation of these existing methods lies in their reliance on traditional methods, operators, and generalizations within the q-ROF framework, which lack the characteristics of the Cq-ROF environment. In comparison, the proposed approach operates within the Cq-ROF framework, which incorporates circular behavior. As a result, existing techniques can be considered special cases of the proposed method. A brief discussion of the limitations and weaknesses of the existing approaches is provided below:

- Liu et al. [31] introduced the WAO and WGO within the framework of q-ROFS, without considering circular behavior. However, the Cq-ROFS framework is a more advanced and generalized form of q-ROFS. Consequently, the work of Liu et al. [31] represents a special case within the broader scope of the proposed Cq-ROFS theory.
- Khan et al. [33] and Ali et al. [34] proposed Aczel–Alsina aggregation operators in the context of q-ROFS, where the information is limited to MD and NMD, without incorporating radius information. As a result, their methods are insufficient for evaluating the data presented in Table 5.
- Farid et al. [35] introduced generalized q-ROFS-based Einstein interactive geometric AOs. Peng et al. [36] explored exponential operations and AOs for q-ROFS. Du et al. [37] developed q-rung orthopair triangular fuzzy Linguistic Frank operators. Jana et al. [38] proposed Dombi aggregation operators for q-ROF numbers. Liu et al. [39] presented q-ROF Heronian mean operators. Notably, all these studies are based on the conventional q-ROFS model, which does not account for circular behavior. Therefore, each of these approaches represents a specific case within the more comprehensive Cq-ROFS framework and is inadequate for handling the data illustrated in Table 5.

Thus, the proposed method presented in this article is a more general and feasible extension of the previous approaches.

9. Conclusion and Future Work

In this article, we introduced the concept of Cq-ROF linguistic term information, which offers a highly reliable and authentic approach within fuzzy set theory and practical real-world decision-making. The integration of the WASPAS method within the Cq-ROFS framework provides several advantages. The proposed AOs and WASPAS model exhibit high flexibility and robustness, enabling effective resolution of MCDM problems and facilitating the modeling of relationships among diverse criteria through adjustable parametric values in the Cq-ROF environment. To validate the practical effectiveness of the proposed methods, we applied them to an experimental case study, identifying the most suitable alternatives for EGDE development. The evaluation was conducted based on linguistic assessments for each criterion using the Cq-ROF-based MCDM technique. To ensure the robustness of our approach, we analyzed the impact of varying the parameter C across different alternative options, confirming the superiority and adaptability of the proposed AOs in comparison to previous methods. This research delivers several important findings:

1. Comprehensive definitions, operational laws, score and accuracy functions, and novel AOs are recalled.
2. Adoption of linguistic terms within the Cq-ROFS framework for both alternative evaluation and criteria importance assessment.
3. Novel idea of Cq-ROFWAO and Cq-ROFGAO, with illustrative examples.
4. Formal proofs of essential properties demonstrating the consistency, reliability, and robustness of the proposed operators and decision-making framework.
5. Prioritization of criteria weights and their application in solving MCDM problems using Cq-ROF information.
6. Implementation of the WASPAS technique within the Cq-ROF framework to aggregate the DM into a unified decision.
7. Application to evaluate strategies for EGDE, demonstrating effective handling of interrelated and uncertain criteria.
8. Assessment of sensitivity analysis and compatibility of the proposed method with existing approaches.

This study effectively addresses inherent uncertainty modeling challenges, yielding a robust evaluation support system that enables accurate and informed decision-making in complex and dynamic environments.

9.1.Constraints and Limitations

Cq-ROF information is highly versatile due to its ability to handle grades beyond the traditional [0,1] interval. The extended WASPAS technique within the Cq-ROF framework offers a sophisticated model for tackling complex real-world problems. However, this model has limitations, such as being restricted to specific information formats (as defined in Definition 3) and struggling with complex fuzzy or bipolar fuzzy information. Additionally, the model requires non-zero criteria weights that sum to 1; otherwise, the results may be inaccurate. Despite these constraints, the proposed model makes a significant contribution to fuzzy set theory, particularly in handling Cq-ROF information.

9.2.Future Direction

This study lays the groundwork for future research that can investigate the integration of diverse AOs, such as Aczel-Alsina, Einstein, and Frank operators, with WASPAS techniques to develop more robust decision-making frameworks. Future studies can also explore alternative methodologies, including TOPSIS, AHP, and MARCOS, to further enrich the initiated work. However, the concept can offer promising avenues for further research, like Aczel–Alsina power AOs [40], Aczel Alsina hamy mean operators [41], Einstein AOs [42], Einstein interactive geometric AOs [43], and Frank AOs [44].

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